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Expectations about Unreported Output, Bank Lending and Double-Cycle Stability Policy

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Abstract

This article argues that the possibility there can be output unreported to the authorities, prompts expectations about the size of this output which can destabilize increasingly an economy experiencing otherwise a uniform oscillation. It follows logically that the "stability" of uniform fluctuations will be preserved if the policy maker aims at such fluctuations in unreported output too, but of exactly opposite direction ("double cycle" hypothesis), lessening in effect the fluctuation of overall output as well. The economy is one modeled in terms of the interplay between its banking sector and the government budget. Our conclusions hold independently of the source of unreported output allowing thus one to identify for analytical convenience this output with everything the term connotes except tax evasion. Assuming that borrower-lender asymmetric information leads to a fraction only of bank lending to be financing capital change, instability becomes a matter of the expectations about this fraction too, about credit rationing; much more so when the capital change involves both sectors of the economy. The link between the two types of expectations is that they are both shaped by the stage of the business cycle, making the "double cycle" target attainable by means of the manipulation of "lending" or the same, credit-rationing expectations. The introduction of money or bank industry structure into the analysis does not appear to alter these conclusions; nor does the examination of the subject in terms of labor in the place of capital− examination enabled analytically through the use of a CES production function.

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1 Introduction

There is a number of papers that study the effect of informal economy on macroeconomic performance. The general conclusion is that "countries with larger informal economies tend to undergo increased volatility in output, investment and consumption over the business cycle" (see e.g. Ferreira-Tiryaki 2008, 91). There is also a consensus on the role that the imperfect measurement of informal economic activity in national accounts plays for assessing the profile or strength of volatility. For example, Restrepo-Echavarría (2014) attributes the observed higher volatility of consumption relative to output to the poor measurement of this economy while Fernández and Meza (2015) emphasize the stronger variability in aggregate economic activity in response to measurement errors. The measurement problem is closely related to the definition of the informal economy. Schneider and Enste (2000) and Williams and Nadin (2010) are a couple only of the classical treatments of the definition debate.

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This paper adopts the convenient definition of "all economic activities that contribute to the officially calculated (or observed) gross national product but are currently unregistered" (Schneider and Enste 2000, 78) in the footsteps of Feige (1994), Frey and Pommerehne (1984), and many others. The terms "informal/unofficial economy", "unreported/unrecorded output", etc. are used below interchangeably in this connection in an examination of how the expectations about the size of informal economy influence macroeconomic stability; expectations formed by businessmen and labor given investment financed partly through bank lending by a banking system used also by the government to be keeping its budget balanced. Part only of investment finance comes from bank and formal in general lending because of the credit rationing induced by borrower-lender asymmetric information. The change of the regime that currently makes some businessmen seek access to resources informally, or many official economy workers believe that the informal wage will go up, are two expectation examples. The institution of banking plays a critical role in providing resources to firms and hence, it should be a key component of the matter under discussion. Banks provide resources to the government as well, and its budget should be a second key component of the analysis.

Methodologically, this paper may be considered to be a formal model of an extended version of the research agenda regarding the link between entrepreneurship and informal economy (Ketchen Jr et al. 2014). The role of expectations is top in the list of this agenda, but it is about microeconomic expectations like those about starting an informal business or working underground. The role of macroeconomic expectations investigated herein is overlooked by this research agenda and is without, yet, any prior formal discussion save that by Dabla-Norris and Feltenstein (2005) where expectations refer to the price level in a general equilibrium model. Indeed, there do exist several formal treatments of macroeconomic performance in the presence of informal economy, but always from the viewpoint of real business cycles models like those reviewed by Granda-Carvajal (2010). Given the general equilibrium character of these models, the system returns back to equilibrium always, and the scope of examining the role of expectations within such an environment is limited. Instead, this paper capitalizes upon the empirical findings about the cyclical relationship between reported and unreported output fluctuations (Giles 1997, Russo 2008), by considering the impact of the subsequent expectations on the "official" business cycle when part of bank lending is channeled to the informal economy.

The next section presents this analysis formally, modeling the bank *à la* Blinder (1987) and assuming a balanced government budget for an unreported output whose source may be anything else than tax evasion-avoidance. Both of these assumptions are made for the mathematical tractability of the modeling; it is also a modeling producing results that are not influenced by the origins of unreported output, which makes it safe theoretically to disregard the source of tax evasion. As a matter of fact, Auriol and Warlters (2005) show empirically that the inverse relationship between shadow economy size and tax revenue ratios vanishes when the endogeneity of the shadow economy is acknowledged. The size of this economy becomes endogenous as a result of tax policy trying to meet specific difficulties in raising tax revenues. For example, many developing countries deliberately create costs of entry into the formal economy in order to reduce competition and create economic rents, which may then be taxed.

Anyway, the basic conclusion is that changing expectations about the size of unreported output destabilize further an economy subject already to uniform oscillations due to the zero-sum between business and government finance, *ceteris paribus*. This is a conclusion speculated through a time-lag approach to capital accumulation, but confirmed by an accelerator approach too, while money and labor appear to follow investment trends passively. The time-lag and accelerator approaches allow one to reach policy conclusions too, by linking the unreported output-induced expectations with the expectations about the fraction of capital change financed by banks, about credit rationing: Policy should be aiming at fostering double cycle conditions as the only way to extend the "stability" of the uniform oscillation to the informal sector of the economy too, and lessen in effect the fluctuation of overall output as well. This, the pursuit of double cycle by manipulating credit rationing expectations, is another means through which the shadow economy may be endogenized beyond the means of tax policy. Section 3 ends this article by expanding on policy issues.

2 Formal Considerations

Let total output, Y, consist of unrecorded and recorded output, Y_u, and Y_v, respectively, so that:

$$Y = Y_u + Y_v \quad (1)$$

Assume that the excess reserves of banks, E, are a decreasing function of Y_v, since it is Y_v that banks observe:

$$E = -m_1 Y_v \quad (2)$$

where m_1 is some positive constant. For example, economic expansion lowers default risks, raising the profitability from lending, and reducing the excess reserves held by banks. In general, bank reserves, R, are:

$$R = E + \rho D \quad (3)$$

or, in view of (2):

$$R = -m_1 Y_v + \rho D \quad (3')$$

where D is bank deposits and $\rho \in [0,1]$ is the required reserves ratio. Also, banks invest in government bonds, and lend firms and hence, the balance sheet identity is:

$$D = R + L + B \quad (4)$$

or, in view of (3):

$$(1-\rho)D = E + L + B \quad (4')$$

where L is lending and B is bond purchases. Certainly, firms may use borrowed funds to cover activities whose output may not be recorded by the authorities. Now, given E, part $z \in (0,1)$ of $(1-\rho)D$ is loaned out to firms and part $1-z$ is used to buy government bonds. Assuming in addition that loans displace bonds as reported economic activity expands and given (2), one obtains that:

$$L = z(1-\rho)D + m_2 Y_v \quad (5)$$

and:

$$B = (1-z)(1-\rho)D - (m_1 + m_2)Y_v \quad (6)$$

since the deals with the government are based on reported only output, too; m_2 is a positive coefficient. Yet, deposits are savings coming out not only from the sector whose output has been recorded but also from the sector with the unrecorded output. That is, if the marginal propensity to consume, s, is the same for the two output types:

$$D = sY \quad (7)$$

Income earned underground can be deposited with a bank most of the time despite the fact that as Tanzi (1980) first noticed, a substantial fraction of the underground activity relies on cash to avoid detection by tax officials. Letting $Y_u = yY_v$, $y \in (0,1)$, and inserting (7) in (5) and (6), yields:

$$L = [z(1-\rho)(1+y)s + m_2]Y_v \quad (5')$$

and:

$$B = [(1-z)(1-\rho)(1+y)s - (m_1 + m_2)]Y_v \quad (6')$$

with y changing, of course, over time, reflecting the relationship between the business cycles in the two sectors of the economy. If, for example, $Y = 100$ and the cycles are exactly opposite to each other ("double cycle", Russo 2008), one obtains that: $Y_u = [y/(1+y)]100$ and $Y_v = [1/(1+y)]100$ always; that is, the two outputs add up to 100 always. In any other case in which one cycle leads or lags, opposite or not, we have for the current time period a $Y_u = [y/(1+y)]100$ combined with a$Y_{v,n} = [1/(1+y_n)]100$, where $n = \pm 1, \pm 2, \ldots$ is time periods ahead $(+)$ or behind $(-)$ the current one, and only by chance the sum of the two outputs will be 100. Figure 1, where $x \equiv y$, depicts $Y_u/100 = [y/(1+y)]$ with the $Y_v/100 = [1/(1+y)]$ of the double cycle and with $Y_v/100 = [1/(1+y-0.2)]$.

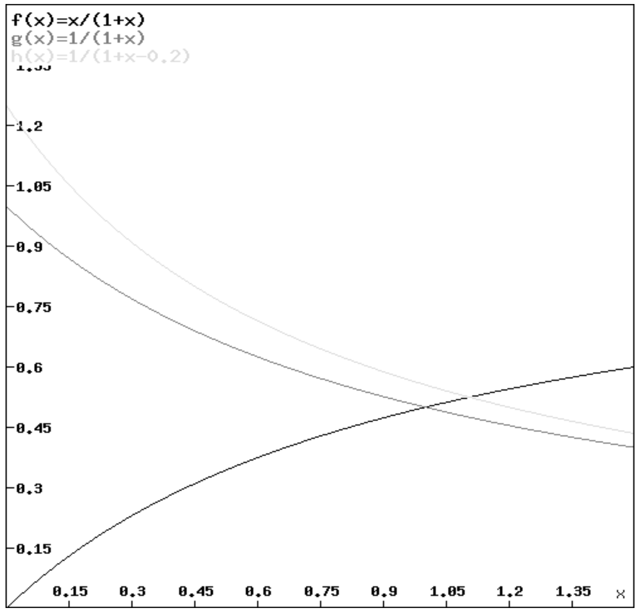

Figure 1: Combination of $Y_u/100$ with two $Y_v/100$

Next, if the government budget constraint is:

$$G = \Delta B + \tau Y_v \qquad (8)$$

inserting the total differential of (6') in (8), gives:

$$G = [(\varphi + \tau)(1 + y) - (m_1 + m_2)]Y_v - [(\varphi + \tau)(1 + y_{-1}) - (m_1 + m_2)]Y_{v,-1} \qquad (9)$$

where G is public expenditure, τ is the average tax rate, Δ denotes change by one time period, the subscript "-1" refers to the previous period, and $\varphi \equiv (1 - z)(1 - \rho)s$. Presumably, $\varphi + \tau > (m_1 + m_2)$. It is assumed that the presence of a positive instead of zero y, the fact and only that there can be an informal economy that can affect overall economic performance, makes the observed value of y to summarize what to expect in terms of performance next period. The presence of a non-zero y affects the stage of the business cycle of the overall economy, and by looking one at the value of the current y, one can sense the current phase of the cycle, and form thereby expectations about next period's phase, which is the same as next period's y, and *vice versa*.

The time-lag approach

Denoting by $(K - K_{-1})$ the change of capital stock from last period, and setting $L = k(K - K_{-1})$, one obtains from (5') that:

$$Y_v = \frac{k(K - K_{-1})}{\psi(1 + y) + m_2} \qquad (10)$$

where $k\epsilon(0,1)$ is the fraction of capital stock change, financed through lending and changing from one time period to the other in response perhaps to economic outlook changes, while $\psi \equiv z(1 - \rho)s$ so that $\varphi + \psi \equiv (1 - \rho)s$. Sectoral capital changes differ, but exist *a priori* in both sectors of the economy, and when a bank lends someone to buy or rent equipment, it might be suboptimal to monitor fully the sectoral use of the loan (Capasso et al. 2015). If not anything else, lending a shadow firm does not necessarily imply that the bank will not get its money back given proper accounting records on the part of bank client.

Nevertheless, borrower-lender asymmetric information, placing a wedge between external and internal business finance, does render k less than the unit. Checking out the accounts of a medium or small enterprise cannot tell the bank if this enterprise has become formal only to seek bank finance (Distinguin et al. 2016). And, banks do know that the taxes prescribed by the tax code for a large corporation which seeks funds from the bank, and the taxes actually paid by this bank client may differ (Hill 2002). Instances like these induce the bank make bank lending more expensive relative to business internal finance by charging a "lemons" at least premium.
Anyway, inserting (10) in (9), yields after some operations the linear second-order difference equation:

$$G = \frac{[(\varphi+\tau)(1+y)-(m_1+m_2)]k}{\psi(1+y)+m_2}K - \frac{2[(\varphi+\tau)(1+y_{-1})-(m_1+m_2)]k_{-1}}{\psi(1+y_{-1})+m_2}K_{-1} + \frac{[(\varphi+\tau)(1+y_{-2})-(m_1+m_2)]k_{-2}}{\psi(1+y_{-2})+m_2}K_{-2}$$

that in terms of the shorthand notation:

$$\Gamma \equiv \frac{[(\varphi+\tau)(1+y)-(m_1+m_2)]k}{\psi(1+y)+m_2}$$
$$\Gamma_{-1} \equiv \frac{[(\varphi+\tau)(1+y_{-1})-(m_1+m_2)]k_{-1}}{\psi(1+y_{-1})+m_2}$$
$$\Gamma_{-2} \equiv \frac{[(\varphi+\tau)(1+y_{-2})-(m_1+m_2)]k_{-2}}{\psi(1+y_{-2})+m_2}$$

produces the steady state solution:

$$\bar{K} = \frac{G}{\Gamma - 2\Gamma_{-1} + \Gamma_{-2}} \qquad (11)$$

which depends critically not only on institutional-policy arrangements as captured by the values of φ, ψ, τ, and (m_1+m_2), but also on previous expectations as reflected through y and k; different y and k produce different $\bar{K}$, *ceteris paribus*. The steady state depends on unreported output expectations, which would not be the case if $y=0$, i.e. if there were no such output, or if $y>0$ and expectations were present, but $y=y_{-1}=y_{-2}$. In any case, the steady state still depends on the expectations about k, and becomes indeterminate when $k=k_{-1}=k_{-2}$ either with $y=0$ or with $y=y_{-1}=y_{-2}>0$: Under these circumstances, the homogeneous equation results in a harmonic oscillation around the long-run trend as described by $K_{1,2}=\left(1\pm i\sqrt{3}\right)/2$, where $i=\sqrt{-1}$; oscillation, due presumably to the zero-sum between business and government finance, *ceteris paribus*
The presence of changing expectations about a non-zero y (and k) is also the reason why the associated homogeneous equation cannot provide clear-cut stability conditions, since what can be made sure out of the solutions:

$$K_{1,2} = \frac{\Gamma_{-1} \pm \sqrt{\Gamma_{-1}^2 - 4\Gamma\Gamma_{-2}}}{2\Gamma_{-2}} \qquad (12)$$

is only that the discriminant is positive: relationship $\bar{K}>0 \Rightarrow \Gamma+\Gamma_{-2}>2\Gamma_{-1} \Rightarrow \Gamma^2+\Gamma_{-2}^2+2\Gamma\Gamma_{-2} > 4\Gamma_{-1}^2$ implies in conjunction with inequality $\Gamma_{-1}^2 > 4\Gamma\Gamma_{-2}$ that $\Gamma^2+\Gamma_{-2}^2+2\Gamma\Gamma_{-2} > 4\Gamma\Gamma_{-2} \Rightarrow \Gamma^2+\Gamma_{-2}^2 - 2\Gamma\Gamma_{-2} = (\Gamma-\Gamma_{-2})^2 > 0$, which is true. Nevertheless, (12) is not algebraically tractable and one cannot tell its value relative to one in order to obtain precise stability conclusions. These results are still valid when either y or k is kept constant. The values of z and ρ do not matter suffices to have a $\tau > (m_1+m_2)$ in case of full reserve banking, $\rho=1$, or of banks that do not lend the government at all, $z=1$. Moreover, it may be easily seen with a little extra algebra that breaking y into its components, for example one part coming out of tax evasion/avoidance, t, and a second part lumping the remaining origins of y together, $y = t + nt$, and considering τ to be the average tax rate after tax evasion, $\tau = t -$ t, it would not alter the results in the least. The introduction of linear progressive taxation would not qualify the results either.

The policy implications of these considerations derive from what the relationship $y = y_{-1} = y_{-2}$ really refers to. Note that these are considerations pertaining to the business cycle of the official economy, and recall that in this case, one can have $y = y_{-1} = y_{-2}$ only under double cycle circumstances. Therefore, adopting from the next subsection the clear-cut conclusion of increasing instability unless $y = y_{-1}(=$ $y_{-2})$, what the policymaker should be pursuing with its policies is a double cycle, two uniform fluctuations opposite to each other with precision. Of course, there is also the matter of the expectations incorporated by k. They too, are influenced by the business cycle and should be related subsequently to y. It follows that $k = k_{-1} = k_{-2}$ when $y = y_{-1} = y_{-2}$, and *vice versa*.

Consequently, the manipulation of k by the government is one policy means through which a double cycle may be attained. Manipulation, really, of credit rationing expectations, of the the wedge between external and internal business finance as circumstances dictate, manipulating at the same time the extent of informal financing through trade credits, intra-industry or family borrowing, etc. (Casey and O'Toole 2014). In Australia, for example, McCarthy et al. (2017, 58) report that "[l]arger small to medium sized enterprises with growth intentions, business plans, and those in the agriculture industry are significantly more likely to seek finance. In contrast, firms in agriculture that are older, and that have incremental product innovation, 40% or more of export sales, and a male Chief Executive Officer, are less likely to be credit rationed." Here, the policymaker has much certainly space to maneuver in terms of the desirable direction of credit rationing expectations.

The accelerator approach

Denoting the desired capital stock by $K^* = aY$ and by $DK = b(K^* - K) = b(aY - K)$ the change of capital stock needed to attain K^*, and setting $L = kDK$, one obtains from (5') that:

$$Y_v = \frac{kbK}{[(kba - \psi)(1 + y) + m_2]} \qquad (13)$$

where $[kba(1 + y)] - [\psi(1 + y) + m_2] > 0$. Sectoral capital changes differ, but exist *a priori* in both sectors of the economy. Consequently, inserting (13) in (9), yields after some operations the linear second-order difference equation:

$$G\frac{[(kba - \psi)(1 + y) + m_2]}{[(\varphi + \tau)(1 + y) - (m_1 + m_2)]kb} = K - \frac{[(\varphi + \tau)(1 + y_{-1}) - (m_1 + m_2)][(kba - \psi)(1 + y) + m_2]k_{-1}}{[(\varphi + \tau)(1 + y) - (m_1 + m_2)][(k_{-1}ba - \psi)(1 + y_{-1}) + m_2]k}K_{-1}$$

that in terms of the shorthand notation:

$$\pi \equiv [(\varphi + \tau)(1 + y_{-1}) - (m_1 + m_2)][(kba - \psi)(1 + y) + m_2]k_{-1}$$
$$\Pi \equiv [(\varphi + \tau)(1 + y) - (m_1 + m_2)][(k_{-1}ba - \psi)(1 + y_{-1}) + m_2]k$$

produces the steady state solution:

$$\bar{K} = \frac{[(kba - \psi)(1 + y) + m_2]\Pi}{[(\varphi + \tau)(1 + y) - (m_1 + m_2)]kb(\Pi - \pi)}G \qquad (14)$$

while the homogeneous equation gives the solution:

$$K = \frac{\Pi}{\pi} \qquad (15)$$

and, since $\overline{K}$ has to be positive, $\Pi - \pi > 0$, one concludes that: $K = \Pi/\pi > 1$, which is the case of instability. A uniform oscillation with $\Pi = \pi$, and hence, indeterminate $\overline{K}$ will be the case when either $y = 0$ or $y = y_{-1}$ and $k = k_{-1}$ always. If $y = 0$ or $y = y_{-1}$, and $k > k_{-1}$, the case of worsening instability case arises again: $\Pi - \pi > 0$. The merit of a policy pursuing a double cycle is now. That is, under this accelerator approach to the topic under examination, the presence of unreported output and changing expectations destabilize economic activity increasingly; yet, instability can be confined to uniform (at best) fluctuations suffices to nullify the role of expectations by means of influencing k. To see how this policy means works, let $k = k_u + k_v$ and $k_u = \kappa k_v$ so that $k = (1+\kappa)k_v$. The total differential of this last equality is: $dk = k_v d\kappa + (1+\kappa)dk_v$. Influencing k, means influencing k_v and hence, $dk/dk_v = 1 + \kappa(1+\eta)$, where $\eta < 0$ is the elasticity of κ with respect to k_v. As before, these conclusions stand independently of the sources of y and under linear progressive taxation.

The "labor" approach

The behavior of labor may be traced too, linking labor and capital through a production function, but this method is viable technically only under the accelerator approach (see Appendix). Algebraic tractability is the reason why sectoral differences in production are assumed away, too. Nevertheless, the complementarity in reported and unreported output consumption (Chiarini and Marzano 2006) does suggest a sort of cars-and-petrol production, which might be seen macroeconomically as taking place within the context of a unique national economy production process. Consider the general case of the CES production function:

$$(1+y)Y_v = [eK^r + (1-e)N^r]^{1/r} \qquad (16)$$

where N is labor in the overall economy, $e \in (0,1)$ is the distribution parameter, and $\sigma = 1/(1-r)$ is the elasticity of substitution. Solving (16) for K and inserting in (13), one obtains that:

$$Y_v = \frac{kb(1-e)}{\left\{kb(1+y)^r - e^{\frac{1}{r}}[(kba-\psi)(1+y)+m_2]\right\}}N$$

which when inserted in turn in (10), gives the following linear first-order difference equation in the labor variable:

$$G\frac{\omega}{\Omega} = N - \frac{\omega\Omega_{-1}}{\omega_{-1}\Omega}N_{-1}$$

where $\omega \equiv \left\{kb(1+y)^r - e^{\frac{1}{r}}[(kba-\psi)(1+y)+m_2]\right\}$, $\omega_{-1} \equiv \left\{k_{-1}b(1+y_{-1})^r - e^{\frac{1}{r}}[(k_{-1}ba - \psi)(1+y_{-1})+m_2]\right\}$, $\Omega \equiv \{kb(1-e)[(\varphi+\tau)(1+y)-(m_1+m_2)]\}$, and $\Omega_{-1} \equiv \{k_{-1}b(1-e)[(\varphi+\tau)(1+y_{-1})-(m_1+m_2)]\}$. The steady state is:

$$\overline{N} = \frac{\omega\omega_{-1}\Omega}{\Omega(\omega_{-1}\Omega - \omega\Omega_{-1})}$$

while the homogeneous equation gives the solution:

$$N = \frac{\omega_{-1}\Omega}{\omega\Omega_{-1}}$$

What is for sure is that $\bar{L}$ is indeterminate and $\omega_{-1}\Omega/\omega\Omega_{-1} = 1$ when $y = y_{-1}$ and $k = k_{-1}$; the same when $y = 0$ and $k = k_{-1}$. This result corroborates conclusions reached earlier in connection with both the accelerator and time-lag approaches. Under a Cobb-Douglas production function, the expectations about k are not involved at all:

$$(1+y)Y_v = K^e N^\epsilon \qquad (17)$$

where ε and ϵ are output elasticities. Solving (17) for K and inserting in (13), one obtains that:

$$Y_v = \left\{\frac{[(kba-\psi)(1+y)+m_2]}{kb(1+y)^{\frac{1}{\varepsilon}}}\right\}^{\frac{\varepsilon}{1-\varepsilon}} N^{\frac{\epsilon}{1-\varepsilon}}$$

which when inserted in turn in (9), gives the following difference equation in the labor variable:

$$G\frac{a}{A} = N^{\frac{\epsilon}{1-\varepsilon}} - \frac{aA_{-1}}{a_{-1}A}N_{-1}^{\frac{\epsilon}{1-\varepsilon}}$$

with steady state:

$$\bar{N} = \frac{aa_{-1}}{a_{-1}A - aA_{-1}}G^{\frac{1-\varepsilon}{\epsilon}}$$

where $\mathrm{A} \equiv [(\varphi+\tau)(1+y)-(m_1+m_2)][(kba-\psi)(1+y)+m_2]^{\frac{\varepsilon}{1-\varepsilon}}$, $\alpha \equiv kb(1+y)^{\frac{1}{1-\varepsilon}}$, $\mathrm{A}_{-1} \equiv [(\varphi+\tau)(1+y_{-1})-(m_1+m_2)][(kba-\psi)(1+y_{-1})+m_2]^{\frac{\varepsilon}{1-\varepsilon}}$, and $a_{-1} \equiv kb(1+y_{-1})^{\frac{1}{1-\varepsilon}}$. Certainly, $a_{-1}A - aA_{-1} > 0 \Rightarrow \bar{N} > 0$. When $y = 0$ or when $y = y_{-1}$, $\bar{N}$ is indeterminate. And, one way to obtain a solvable homogeneous equation is to set: $N_{-1}^{\frac{\epsilon}{1-\varepsilon}} = x$, in which case:

$$N = \left(\frac{a_{-1}A}{aA_{-1}}\right)^{\frac{1-\varepsilon}{\epsilon}}$$

Since, $\bar{N} > 0 \Rightarrow a_{-1}A > aA_{-1}$, increasing instability will be the case unless $y = 0$ or when $y = y_{-1} > 0$, because then $a_{-1}A = aA_{-1}$, and we have a uniform oscillation. It is noteworthy that production technology considerations do not alter these conclusions at all. It is also notable that no special labor policy is required to attain the double business cycle, which continues to be a matter of the manipulation of k.

The monetary approach

Let us next introduce the total stock of money, M, which consist of the stocks employed in the two sectors of the economy:

$$M = M_u + M_v$$

Set, in the spirit of Werner (2012):

$$M = \theta Y_u + \vartheta Y_v$$

where θ and ϑ might be thought of as Pigou constants; we have implicitly a monetary authority adjusting money supply to meet money demand, which is much higher in the unreported output sector (Pickhardt and Sardà 2011). Hence,

$$Y_v = \frac{M}{\vartheta + \theta y} \qquad (18)$$

which, of course, has to be equal to (10) and (13). Inserting (18) in (9), yields:

$$G\frac{(\vartheta+\theta y)}{[(\varphi+\tau)(1+y)-(m_1+m_2)]}=M-\frac{(\vartheta+\theta y)[(\varphi+\tau)(1+y_{-1})-(m_1+m_2)]}{(\vartheta_{-1}+\theta_{-1}y_{-1})[(\varphi+\tau)(1+y)-(m_1+m_2)]}M_{-1}$$

with steady state:

$$\bar{M}=\frac{(\vartheta+\theta y)(\vartheta_{-1}+\theta_{-1}y_{-1})}{\Lambda-\Xi}G$$

where: $\Lambda\equiv(\vartheta_{-1}+\theta_{-1}y_{-1})[(\varphi+\tau)(1+y)-(m_1+m_2)]$ and $\Xi\equiv(\vartheta+\theta y)[(\varphi+\tau)(1+y_{-1})-(m_1+m_2)]$. If $y=0$:

$$\bar{M}=\frac{\vartheta\vartheta_{-1}}{(\vartheta_{-1}-\vartheta)[\varphi+\tau-(m_1+m_2)]}G \qquad (19)$$

which can be positive only if $\vartheta_{-1}>\vartheta$. If $y=0$ and $\vartheta_{-1}=\vartheta$, $\bar{M}$ becomes indeterminate. If $y>0$, $\vartheta_{-1}=\vartheta$, and $\theta_{-1}=\theta$,

$$\bar{M}=\frac{(\vartheta+\theta y)}{(\varphi+\tau)(y-y_{-1})}G$$

which too, becomes indeterminate if $y=y_{-1}$. The homogeneous equation gives the solution:

$$M=\frac{(\vartheta_{-1}+\theta_{-1}y_{-1})[(\varphi+\tau)(1+y)-(m_1+m_2)]}{(\vartheta+\theta y)[(\varphi+\tau)(1+y_{-1})-(m_1+m_2)]} \qquad (20)$$

which is impossible technically to compare with one and arrive at safe stability conclusions. If $y=0$:

$$M=\frac{\vartheta_{-1}}{\vartheta}$$

which will exceed one if $\vartheta_{-1}>\vartheta$ as argued earlier in connection with (20), this has to be the case in order to have $\bar{M}>0$. That is, if $\bar{M}>0$, it will be destabilizing even if K oscillates harmonically. Since it cannot be that $\bar{M}<0$ and hence, that $\vartheta_{-1}<\vartheta$, only the case $\vartheta_{-1}=\vartheta$ and indeterminate thus $\bar{M}$ emerges as the one consistent with uniform fluctuation of K around an indeterminate $\bar{K}$ under the time-lag or accelerator approaches. If $y>0$, $\vartheta_{-1}=\vartheta$, and $\theta_{-1}=\theta$, (21) cannot be compared to one safely unless in addition: $y=y_{-1}$, in which case $M=1$: We have a uniform oscillation of M and indeterminate $\bar{M}$ again. The remark that these results are independent of the origins of y and hold when taxation is linearly progressive, applies here, too. Policy-wise, since ϑ and θ are influenced by y, the pursuit of $y=y_{-1}$ in the realm of a double cycle, does not require any special attention to monetary policy beyond that in connection with the manipulation of k.

The manipulation of central bank money is not necessary as may be seen by letting, for example, money may be used to finance G as well.

$$G=\Delta B+\tau Y_v+\Delta M \qquad (8')$$

$$G=[\varphi+\tau-(m_1+m_2)]\left(Y_v-Y_{v,-1}\right)+(\varphi+\tau)\left(yY_v-y_{-1}Y_{v,-1}\right)+(\vartheta+\theta y)Y_v-(\vartheta_{-1}+\theta_{-1}y_{-1})Y_{v,-1} \qquad (9')$$

Inserting (13) in (9'), one obtains that:

$$G\frac{(\vartheta+\theta y)}{[(\varphi+\tau)(1+y)-(m_1+m_2)+(\vartheta+\theta y)]}=M-\frac{(\vartheta+\theta y)[(\varphi+\tau)(1+y_{-1})-(m_1+m_2)+(\vartheta_{-1}+\theta_{-1}y_{-1})]}{(\vartheta_{-1}+\theta_{-1}y_{-1})[(\varphi+\tau)(1+y)-(m_1+m_2)+(\vartheta+\theta y)]}M_{-1}$$

The only algebraically tractable case is the one according to which, $\vartheta=\vartheta_{-1},\theta=\theta_{-1}$, and $y=y_{-1}$ and hence, $M=M_{-1}$. Exogenous money is passive in a setting where the protagonist of developments is credit money.

The bank-industry approach

The double-cycle policy suggestion is immune also to bank industry considerations as follows. Let the bank be the single product firm with production function:

$$Q = D^{\zeta} L^{\xi} B^{\phi}$$

or, inserting (7), (5'), and (6') and solving for Y_v:

$$Y_v = \left\{\frac{Q}{[(1+y)s]^{\zeta}[\psi(1+y)+m_2]^{\xi}\big[[\varphi(1+y)-(m_1+m_2)]\big]^{\phi}}\right\}^{\frac{1}{\varsigma}} \qquad (21)$$

where Q stands for bank services produced while ζ, ξ, and ϕ are positive coefficients whose sum captures the economies of scale, ς. Inserting next (21) in (9) yields that:

$$G = \frac{[(\varphi+\tau)(1+y)-(m_1+m_2)]Q^{\frac{1}{\varsigma}}}{\left\{[(1+y)s]^{\zeta}[\psi(1+y)+m_2]^{\xi}\big[[\varphi(1+y)-(m_1+m_2)]\big]^{\phi}\right\}^{\frac{1}{\varsigma}}} - \frac{[(\varphi+\tau)(1+y_{-1})-(m_1+m_2)]Q_{-1}^{\frac{1}{\varsigma}}}{\left\{[(1+y_{-1})s]^{\zeta}[\psi(1+y_{-1})+m_2]^{\xi}\big[[\varphi(1+y_{-1})-(m_1+m_2)]\big]^{\phi}\right\}^{\frac{1}{\varsigma}}}$$

Letting:

$$\mathcal{F} \equiv \frac{[(\varphi+\tau)(1+y)-(m_1+m_2)]}{\left\{[(1+y)s]^{\zeta}[\psi(1+y)+m_2]^{\xi}\big[[\varphi(1+y)-(m_1+m_2)]\big]^{\phi}\right\}^{\frac{1}{\varsigma}}}$$

while $\mathcal{F}_{-1}$ is the same as $\mathcal{F}$ is but with the y replaced by y_{-1}, one obtains the steady state:

$$\bar{Q} = \left(\frac{G}{\mathcal{F}-\mathcal{F}_{-1}}\right)^{\varsigma} \qquad (22)$$

and the homogeneous equation gives the solution:

$$Q = \left(\frac{\mathcal{F}}{\mathcal{F}_{-1}}\right)^{\varsigma} \qquad (23)$$

At steady state, $y = y_{-1}$, which implies that $\mathcal{F} = \mathcal{F}_{-1}$, and the long-run equilibrium is indeterminate. In so far as stability in connection with (23) is concerned, if $\mathcal{F}-\mathcal{F}_{-1} > 0 \Leftrightarrow \mathcal{F}/\mathcal{F}_{-1} > 1$ when $y \gtreqless y_{-1}$ then $\mathcal{F}-\mathcal{F}_{-1} < 0 \Leftrightarrow \mathcal{F}/\mathcal{F}_{-1} < 1$ when $y \lesseqgtr y_{-1}$, which cannot be the case. So, the only viable alternative is once again that of $y = y_{-1}$, of a uniform cycle around an indeterminate steady state and does not depend on bank-industry structure because the origin of Q in (22) and (23) may be any such structure.

For example, consider the market equilibrium of steady state, where supply equals demand. According to Klette and Griliches (1996), a monopolistically competitive bank industry output may be approached as a deflated weighted average of individual bank revenue, R, with the weights μ being market shares:

$$Q = \frac{\sum \mu R}{p_I}$$

where p_I is the industry price. Hence, (22) may be rewritten as:

$$\frac{\sum \bar{\mu}\bar{R}}{\bar{p}_I} = \left(\frac{G}{\mathcal{F} - \mathcal{F}_{-1}}\right)^{\varsigma} \qquad (22')$$

where the bars above μ, R and p_I are those associated with the long-run monopolistically competitive equilibrium of steady state. Rewriting Q as in (22') does not change any conclusions. Or, consider the Cournot paradigm:

$$Q = \frac{\mathcal{M}}{\mathcal{M}+1}\hat{Q}$$

where $\mathcal{M}$ is the number of banks, and $\hat{Q}$ is the output of perfectly competitive banking. (22) becomes:

$$\frac{\mathcal{M}}{\mathcal{M}+1}\hat{Q} = \left(\frac{G}{\mathcal{F} - \mathcal{F}_{-1}}\right)^{\varsigma} \qquad (22'')$$

and nothing else changes. In any case, immaterial for our results are the economies of scale as well. Indeed, as Beck et al. (2011) put it, "the evidence suggests that the lending environment is more important than firm size or bank ownership type in shaping bank financing to SMEs".

3 Concluding Remarks

It has been established that stabilization in the presence of unreported output amounts to the pursuit of a double cycle as a policy target attainable be means of manipulating k, that is, bank lending expectations. It is a result immune to production technology considerations, origin of unreported output, progressiveness of the tax system, central bank money issuance, and bank industry structure. Authors like Gobbi and Zizza (2007, 1) might question the empirical soundness of the bank sector policy proposed above on the grounds that the "effects of financial deepening on the size of the informal sector are weak and statistically not significant".

Nevertheless, according to Capasso et al. (2015, 1) there does exist a significant link between banks and shadow economy because "as the aggregate level of informality and tax evasion increase, it becomes more profitable for banks to screen and supervise borrowers using more costly in-depth monitoring technologies." They reach this conclusion looking at firms as bank clients and not at workers as Gobbi and Zizza (2007) do. Hasan et al. (2014, 1) establish the role of banks in shaping the underground economy from the viewpoint that "[f]irms with greater tax avoidance also incur more stringent non-price loan terms... and prefer bank loans over public bonds when obtaining debt financing." In sum, the proposed "double-cycle policy target/lending-expectations policy instrument plus perhaps the instrument of public debt" scheme does have as much real-world relevance in endogenizing the shadow economy as tax policy does.

Moreover, these considerations raise the issue of the cooperation between microeconomic policies in connection with the informal economy and the macroeconomic policy suggested herein. The microeconomic approach to the informal economy provides an understanding of the diverse sector activities, the difference in their development, public revenue losses, and misrepresentation of the main economic indicators guiding policy, which comes to add to the difficulties in measuring the informal economy as a whole. The subsequent microeconomic policy measures should be implemented in accordance with the pursuit of the double cycle target and not as having merit *per se*. For example, the ratio of corporate taxes to GDP is pro-cyclical, rising in expansions and declining in contractions (Hill

2002). Tax law revisions and regulation should be pursued with an eye towards the set of banking sector measures that would deal with this matter in the realm of the double cycle policy.

Finally, let us not forget that our analysis includes unreported income from criminal activities: drugs, prostitution, theft, fraud, smuggling/trafficking, illegal gaming/gambling, and welfare program abuse to name a few. Ardizzi et al. (2012) report, for instance, that the average Italian unobserved economy between 2005 and 2008 was 16.5%+10.9% = 27.4% of the total economy, with the 10.9% coming out of illegal activities. The moral mandate of cracking down on such activities should not be compromised for the sake of macroeconomic benefits. Macroeconomic policy should be designed is by taking for granted that police has to be doing its "job" as best as possible. To the extent that poverty nourishes crime, the double cycle policy does help police in its job, because it does not allow poverty-based informality to survive long enough to prompt a poverty trap and embed criminality in society. This "indirect" effect of the proposed policy on a quantity that cannot be put into the control of credit rationing, applies in general to all those informal activities seeking informal finance. In do far as further research on the subject is concerned, it is clear that a detailed elaboration of the recommended policy target/instrument is required.

Appendix

To see the behavior of labor within the realm of the time-lag approach is not possible mathematically. Consider at first the general case of a CES production function as it appears in the text. Solving it for K and inserting the result in (11), one obtains that:

$$Y_v^r = -\frac{(1-e)(N^r - N_{-1}^r) + (1+y_{-1})^r Y_{v,-1}^r}{\{e[\psi(1+y)+m_2]^r - k(1+y)^r\}}$$

which can be positive if either: $e[\psi(1+y)+m_2]^r - k(1+y)^r > 0$ and $(N^r - N_{-1}^r) < 0$, or: $e[\psi(1+y)+m_2]^r - k(1+y)^r < 0$ and $(N^r - N_{-1}^r) > 0$. Any further pursuit of the matter algebraically becomes meaningless because the algebra becomes quite complex. For example, Y_v becomes a function of past Y_v's:

$$Y_{v,-1}^r = -\frac{(1-e)(N_{-1}^r - N_{-2}^r) + (1+y_{-2})^r Y_{v,-2}^r}{\{e[\psi(1+y_{-1})+m_2]^r - k(1+y_{-1})^r\}},$$
$$Y_{v,-2}^r = -\frac{(1-e)(N_{-2}^r - N_{-3}^r) + (1+y_{-3})^r Y_{v,-3}^r}{\{e[\psi(1+y_{-2})+m_2]^r - k(1+y_{-2})^r\}},$$

$Y_{v,-3}^r = \cdots$, and so on. The same problem with the time-lag approach arises even if a Cobb-Douglas production function is adopted, since then one obtains that:

$$Y_v = \frac{k\left[(1+y_{-1})Y_{v,-1}\right]^{\frac{1}{\varepsilon}}}{\left\{\left[k[(1+y)]^{\frac{1}{\varepsilon}} - \psi(1+y) + m_2\right]\right\}} \frac{N^{\frac{\epsilon}{\varepsilon}}}{N_{-1}^{\frac{\epsilon}{\varepsilon}}}$$

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