Bulletin of Applied Economics, 2020, 7(1), 95-105

Productivity, efficiency and firm's market value: Microeconomic evidence from multinational corporations

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Abstract

The paper proposes a conditional range directional distance estimator by modifying the range directional distance model utilizing the probabilistic characterization of directional distance functions (DDF). Moreover, as an illustrative example the paper applies the proposed estimator on a sample of 89 multinational corporations for the period 2006-2012. The paper examines the effect of firms' market value on their estimated operational performance levels. Inefficiency measures are estimated over the examined period. The results reveal a nonlinear (U-shape) relationship between firms' market value and their operating efficiency levels. Finally, the analysis from applying the local linear estimator reveals that lower market values are associated with higher operating inefficiencies, whereas, higher market values are associated with higher operating efficiencies.

JEL classification numbers: C14, D24, M21
Keywords: Productivity, Firm' production, Efficiency, Market value, Microeconomic analysis.

1 Introduction

The directional distance function (DDF) is a popular methodological tool for the measurement of efficiency and productivity of decision making units (Luenberger 1992, 1994; Luenberger 1992, 1994). The main advantage of such methodological tools is its flexibikity to deal with different types of data (among others Färe et al., 1989, 2006, 2007a, 2007b; Färe and Grosskopf, 2009; Chung et al., 1997; Kuosmanen, 2005; Kuosmanen and Podinovski, 2009; Zelenyuk, 2013).

Moreover different estimators based on directional distance directional distance functions have been applied recently from several studies. For instance Asmild and Pastor (2010) have extended the Multi-directional efficiency analysis (MEA) model presented by Bogetoft and Hougaard (1999) and the Range Directional Model (RDM) presented by Silva Portela et al. (2004) in order to account for any type of technical inefficiency. Analysing the Japanese banking industry Barros et al. (2012) have used a non-radial directional distance measurement in treating nonperforming loans as bad output. In their study Akther et al. (2013)analyse the Bangladesh banking industry by applying a two stage slack based directional distance function model. Finally, Chen et al. (2013) present a super efficiency directional distance model suitable to address infeasibility issues under the variable returns to scale assumption (VRS).

Nonetheless all of the pre-mentioned DDF studies address successfully the inefficient measurement under different research conditions, however, they do not account for environmental

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Article Info: *Received*: January 11, 2020 *Published online*: February 20, 2020

(exogenous to the production process) factors characterizing the operating environment (Avkiran, 2009; Ramanathan and Yunfeng, 2009) of the evaluated decision making unit (DMU). According to the DEA literature three are the main dominant approaches which are applied for the analysis of the effect of the environmental factors on the derived efficiency levels and therefore to explain the efficiency variations of the DMUs. These are the double bootstrap approach (Simar and Wilson, 2007), the probabilistic approach (Daraio and SImar, 2005, 2006, 2007) and the semi-nonparametric StoNED (stochastic semi-nonparametric envelopment of data) approach (Johnson and Kuosmanen 2011, 2012; Kuosmanen, 2012).

Recently Simar and Vanhems (2012) based on the probabilistic approach developed the probabilistic characterization of directional distance function, thus enabling to incorporate directly into the measurement the effect of an exogenous (environmental) factor on the derived inefficiency measures. The main advantage of the probabilistic approach is that does not impose a separability assumption. Therefore it can be assume that the exogenous variable(s) directly influence the shape of the frontier (i.e., a separability condition does not hold). As a result, the obtained inefficiency estimates are determined by the inputs, outputs and the exogenous variable(s). This paper applies the probabilistic approach DDF approach in order to modify the RDM estimator (Silva Portela et al., 2004) to be able to account for environmental variables. Since the RDM estimator has appealing and distinct features to the researchers[2] this paper develops a conditional range directional distance (CRDD) estimator. Finally an empirical application of the estimator is presented evaluating the effect of market value on 89 global firms' inefficiency levels over the period 2006-2012. The rest of this paper unfolds as follows. In Section 2 the CRDD estimator is analysed and presented. Section 3 presents the empirical application and a brief literature of the theoretical background linking the exogenous variable (firms' market value in this case) and firms' operating efficiency levels. Finally, Section 4 concludes the paper.

2 Methodology

2.1 The range directional distance function

Let the input quantities of the examined firms (decision making units-DMUs) to be represented by an input vector $p = (p_1, \dots, p_N) \in \mathbb{R}_+^N$ and an output vector $q = (q_1, \dots, q_M) \in \mathbb{R}_+^M$. Then the data of an observed firm $\xi \in (1, \dots, \Xi)$ are represented by the vector $\left(p^\xi, q^\xi\right)$.Then the production set Γ represents the production set of all technically feasible combinations of firms' inputs and outputs. This can be formally defined as:

$$\Gamma = \left\{(p, q) \in \mathbb{R}_+^{N+M} \middle| p \text{ can produce } q\right\} \tag{1}$$

or alternative can be defined by the output set as:

$$F(p) = \{q | (p, q) \in \Gamma\}, p \in \mathbb{R}_+^N \ . \tag{2}$$

Based on Farrell (1957) the output technical efficiency corresponding to a firm at a point $(p, q) \in \mathbb{R}_+^{N+M}$ is given by:

$$\varphi(p, q) = \sup\{\varphi > 0 | (p, \varphi q) \in \Gamma\}. \tag{3}$$

Moreover, following Chambers et al. (1996,1998) the directional distance function for firms with a direction vector $d = \left(-d_p, d_q\right)$ can be defined as:

$$\Delta\left(p^\xi, q^\xi; d_p, d_q\right) = sup\left\{\gamma > 0 | p^\xi - \gamma d_p, q^\xi + \gamma d_q \in \Gamma\right\} \tag{4}$$

However since it is considered here the output directional distance functions, then $-d_p = 0$ and $d_q = q$, therefore (4) becomes:

$$\Delta\left(p^\xi, q^\xi; 0, d_q\right) = sup\left\{\gamma > 0 | p^\xi, q^\xi + \gamma d_q \in \Gamma\right\}. \tag{5}$$

Furthermore under the direction $(0, q)$ then the output radial distance measure defined in (3) can be retrieved. The directional distance function measures firms' inefficiency levels by taking

[2]The RDM estimator is able to handle negative data, is units and translation invariant and the point of reference which the efficiency is measured is not the origin but an "ideal point" based on the inputs/outputs of the sample under examination.

values $\Delta(p^{\xi}, q^{\xi}; 0, d_q) \geq 0$. Moreover if $\Delta(p^{\xi}, q^{\xi}; 0, d_q) = 0$ then the firm under consideration is efficient (zero inefficiency) since it lies on the frontier.

Using data envelopment analysis (DEA) equation (5) for a firm ξ can be estimated as[3]:

$$\hat{\Delta}\left(p^{\xi}, q^{\xi}; 0, d_q\right) = \max \gamma\, ; s.t. \sum_{\xi=1}^{\Xi} \eta_{\xi} q_{\xi m} \geq q_{\xi m} + \gamma d_{qm}, m = 1, \dots M; \sum_{\xi=1}^{\Xi} \eta_{\xi} p_{\xi n} \leq p_{\xi n}, n = 1, \dots, N; \sum_{\xi=1}^{\Xi} \eta_{\xi} = 1, \eta_{\xi} = 0, \xi = 1, \dots, \Xi. \quad (6)$$

Silva Portela et al. (2004) proposed a range directional model (RDM) where the output directional distance function in (5) can be presented as:

$$\Delta(p^{\xi}, q^{\xi}; 0, S_m) = sup\{\gamma > 0 | p^{\xi}, q^{\xi} + \gamma S_m \in \Gamma\}, \quad (7)$$

where the output direction $S_m$for a firm ξ can be defined as:

$$S_{\xi m = \max\{q_{\xi m}; \xi = 1, \dots, \Xi\} - q_{\xi m}, (m = 1, \dots M)} \quad (8)$$

The DEA program for the output RDM can then be obtained as:

$$\hat{\Delta}\left(p^{\xi}, q^{\xi}; 0, S_m\right) = \max \gamma\, ; s.t. \sum_{\xi=1}^{\Xi} \eta_{\xi} q_{\xi m} \geq q_{\xi m} + \gamma S_{\xi m}, m = 1, \dots M; \sum_{\xi=1}^{\Xi} \eta_{\xi} p_{\xi n} \leq p_{\xi n}, n = 1, \dots, N; \sum_{\xi=1}^{\Xi} \eta_{\xi} = 1, \eta_{\xi} = 0, \xi = 1, \dots, \Xi. \quad (9)$$

The output vector $S_{\xi m}$ refers to the range of possible improvement in the output direction of firm ξ. The range vector imposed on the outputs satisfies the non-negativity restriction on the direction vector. Moreover and strictly under the assumption of VRS the DEA model in (9) has been proven to be translation invariant and unit invariant Silva Portela et al. (2004, pp. 1113-1114)[4].

2.2 The conditional directional distance function

Following the work by Cazals et al. (2002), Daraio and Simar (2005, 2006, 2007) introduced the probabilistic characterization of the production process[5]. Let us define the joint probability function of $\Lambda_{P,Q}(.,.)$ as:

$$\Lambda_{P,Q}(p, q) = \text{Prob}(P \leq \text{p}, \text{Q} \geq \text{q}). \quad (10)$$

Then the following decomposition can be obtained:

$$\Lambda_{P,Q}(p, q) = \text{Prob}(P \leq \text{p} | \text{Q} \geq \text{q}) \text{Prob}(P \leq \text{p}) = T_{Q|P}(q|p) X_P(p), \quad (11)$$

where $X_P(p) = \text{Prob}(P \leq \text{p})$ and $T_{Q|P}(q|p) = \text{Prob}(P \leq \text{p} | \text{Q} \geq \text{q})$

Let us have $\omega = (\omega_1, \dots, \omega_r) \in \mathbb{R}^r$ indicates the external variable (or the exogenous factor) to the specified production process. Then equation (10) becomes:

$$\Lambda_{P,Q|\Omega}(p, q|\omega) = \text{Prob}(P \leq \text{p}, \text{Q} \geq \text{q} | \Omega = \omega), \quad (12)$$

which complete characterizes production process. Then, in the same lines to Daraio and Simar (2005, 2006, 2007), the following decomposition can be derived:

$$\Lambda_{P,Q|\Omega}(p, q|\omega) = \text{Prob}(P \leq \text{p} | \text{Q} \geq \text{q}, \Omega = \omega) \text{Prob}(\text{Q} \geq \text{q}) = T_{Q|P,\Omega}(q|p, \omega) X_{P|\Omega}(p|\omega). \quad (13)$$

Then the estimator of the conditional survival function introduced above can be obtained from:

$$\hat{T}_{Q|P,\Omega}(q|p, \omega) = \frac{\sum_{i=1}^{n} I(\text{Q} \geq \text{q}, P \leq \text{p}) K_h(\Omega_i, \omega)}{\sum_{i=1}^{n} I(P \leq \text{p}) K_h(\Omega_i, \omega)}, \quad (14)$$

[3]The linear programming in (6) measures the output directional distance function under the assumption of variable returns to scale (VRS). The reason why the estimator is under the assumption of VRS will be apparent later.

[4]Amild and Pastor (2010) provide an illustrative example presenting evidence that the RDM under the constant returns to scale (CRS) lacks of translation invariance. However they extend the RDM model under the CRS assumption. Lately Färe and Grosskoph (2013) have proved that the CCR score is invariant if only if the data are multiplicatively transformed. In addition they have proved that the DEA-VRS directional distance function score is invariant only if the data are affinely transformed.

[5]For the theoretical background and the asymptotic properties of nonparametric conditional efficiency measures see Jeong et al. (2010).

where $K_h(\Omega_i, \omega) = h^{-1}K((\Omega_i, \omega)/h)$ with $K(.)$ being a univariate kernel defined on a compact support (Epanechnikov in this case) and h is the appropriate bandwidth calculated following Bădin et al. (2010)[6]. Finally, Simar and Vanhems (2012) introduced the probabilistic version of the output directional distance function as (analogue to equation 5):

$$\Delta(p^{\xi}, q^{\xi}; 0, d_q) = sup\{\gamma > 0 | \Lambda_{P,Q}(p^{\xi}, q^{\xi} + \gamma d_q) > 0\}. \quad (15)$$

Then the conditional form can be presented as:

$$\Delta(p^{\xi}, q^{\xi}; 0, d_q | \omega) = sup\{\gamma > 0 | \Lambda_{Q|P,\Omega}(q^{\xi} + \gamma d_q, p^{\xi} | \Omega = \omega) > 0\}. \quad (16)$$

2.2 The conditional range directional distance function

Following equations (7) and (15) the proposed probabilistic form of the output range directional distance function can presented as:

$$\Delta(p^{\xi}, q^{\xi}; 0, S_m) = sup\{\gamma > 0 | \Lambda_{P,Q}(p^{\xi}, q^{\xi} + \gamma S_m) > 0\}, \quad (17)$$

and additionally the proposed conditional range directional distance function (CRDD) can be obtained as:

$$\Delta(p^{\xi}, q^{\xi}; 0, S_m | \omega) = sup\{\gamma > 0 | \Lambda_{Q|P,\Omega}(q^{\xi} + \gamma S_m, p^{\xi} | \Omega = \omega) > 0\}. \quad (18)$$

The linear programming estimating the proposed CRDD function for a firm $\acute{k}$ is presented below:

$$\hat{\Delta}\left(p^{\xi}, q^{\xi}; 0, S_m \middle| \omega\right) = \max \gamma\, ; s.t. \sum_{\xi=1,\dots,\Xi||\Omega_{\xi}-\omega|\leq h} \eta_{\xi} q_{\xi m} \geq q_{\xi m} + \gamma S_{\xi m}, m=1,\dots M; \sum_{\xi=1,\dots,\Xi||\Omega_{\xi}-\omega|\leq h} \eta_{\xi} p_{\xi n} \leq p_{\xi n}, n = 1, \dots, N; \sum_{\xi=1,\dots,\Xi||\Omega_{\xi}-\omega|\leq h} \eta_{\xi} = 1, \eta_{\xi} = 0, \xi = 1, \dots, \Xi \text{ such that } |\Omega_{\xi} - \omega| \leq h. \quad (19)$$

As previously stated when $\hat{\Delta}\left(p^{\xi}, q^{\xi}; 0, S_m \middle| \omega\right) = 0$ indicates and efficient DMU (0 inefficiency) whereas $\hat{\Delta}\left(p^{\xi}, q^{\xi}; 0, S_m \middle| \omega\right) > 0$ indicates inefficiency. However these inefficiency measures capture the effect of the exogenous variable (ω) since it is assumed that influences directly the shape of the estimated production frontier. Therefore the conditional range directional distance function is obtained only by points taking their Ω value in the neighbourhood of ω (Daraio and Simar, 2007). Therefore, as can be realised by the LP in (19) the most crucial part of the estimation is the smoothing parameter or the bandwidth (h) which can be calculated following the Least Squares Cross Validation based approach introduced by Bădin et al. (2010). Moreover, the proposed CRDD estimator in (19) is translation invariant and units invariant (see Appendix for proofs).

2.2 Measuring the effect of the exogenous variable

As second stage analysis we identify the effect of the exogenous variables (Ω) on firms' efficiency levels without specifying in prior any functional relationship as has been proposed by Daraio and Simar (2005, 2006, 2007) . When Ω is univariate (as in our case), a scatter plot of the ratio $Q = \frac{1+\hat{\Delta}\left(p^{\xi}, q^{\xi}; 0, S_m \middle| \omega\right)}{1+\hat{\Delta}\left(p^{\xi}, q^{\xi}; 0, S_m\right)}$ against Ω and its smooth nonparametric regression line would be able to describe the effect of Ω on firms' efficiency levels.

In this study by following Li and Racine (2005) and Racine and Li (2004) the local linear estimator is applied. This will have respectively the form:

$$Q_i = \alpha + \acute{\beta}(\Omega_i - \omega) + e_i \quad (20)$$

Given that Q_i can be firm's i ratio of conditional to unconditional efficiency measures let Ω_i be the external variable, then by using the $\Omega_i - \omega$ instead of Ω_i the intercept equals to $E(Q_i | \Omega_i = \omega)$. If we fit the linear regression through the observations $|\Omega_i - \omega| \leq h$ this can be written as:

$$\min_{\alpha,\beta} \sum_{i=1}^{n} \left(Q_i - \alpha + \acute{\beta}(|\Omega_i - \omega|)\right)^2 I(|\Omega_i - \omega| \leq h) \quad (21)$$

[6] The calculation of bandwidth by Bădin et al. (2010) is based on the Least Squares Cross Validation (LSCV) criterion introduced by Hall et al. (2004) and Li and Racine (2007).

or setting $\varpi_i = \frac{1}{|\Omega_i - \omega|}$ then we have the explicit expression of:

$$\binom{\hat{\alpha}(\omega)}{\hat{\beta}(\omega)} = \left(\sum_{i=1}^{n} I(\|\Omega_i - \omega\| \leq h)\varpi_i \acute{\varpi}_i\right)^{-1}\left(\sum_{i=1}^{n} I(\|\Omega_i - \omega\| \leq h)\varpi_i Q_i\right) = \left(\sum_{i=1}^{n} K\left(H^{-1}(\Omega_i - \omega)\right)\varpi_i \acute{\varpi}_i\right)^{-1}\left(\sum_{i=1}^{n} K\left(H^{-1}(\Omega_i - \omega)\right)\varpi_i Q_i\right). \quad (22)$$

The kernel function is represented by $K(.)$ And the bandwidth by h which is calculated utilizing the least squares cross-validation data driven method.

Finally, since the output conditional and unconditional distance functions are constructed, following the interpretation by Daraio and Simar (2005, 2006, 2007), an increasing regression line between Q and Ω signifies a favorable effect, where as a decreasing line will indicate a negative.

3 An empirical application

3.1 A brief literature and description of variables

According to Modigliani and Miller (1958) one of the firms' rational of decision making is the maximization of their market value[7]. This involves the acquisition of an asset if only if it will increase its owner equity. Therefore an asset it is purchased only if it adds more to the firms' market value than the costs of its acquisition. Hughes et al. (1997) suggest that market value should reflect firms' efficiency levels. By analyzing the market value inefficiency levels of a sample of 190 publicly traded bank holding companies, they suggest that banks' lost market value is strongly connected with banks' production inefficiency levels. Seiford and Zhu (1999) in their study suggest that market value is a measure of evaluating stock marketability. In their analysis of 55 U.S. commercial banks, link banks' market value with their operational efficiency levels in a two stage DEA model. Seiford and Zhu's DEA model has been used by Luo (2003) analyzing a sample of 245 large banks by linking operational efficiency with market efficiency. Finally, more recently in two stage DEA framework, Wang et al. (2013) link firms' market value with their operational efficiencies analyzing a sample of 65 high-technology industry firms for the period 2003-2007. In the corporate finance literature, market value is a key variable which is associated with firms' efficiency levels (Mork et al., 1988; McConnell and Servaes, 1990; Mehran, 1995). More specifically in most of the corporate finance studies Tobin's Q ratio is used as a proxy of firm's performance[8] making therefore a direct association of firm's market value with their operational efficiency levels. As such in this study market value is used as an exogenous value which is assumed that influences firms' inefficiency levels. Finally, based on the existing literature it is expected that firms' lower inefficiency levels to be associated with higher market values and vice versa.

Table 1: Descriptive statistics of the variables used

	2006	*2007*	*2008*	*2009*	*2010*	*2011*	*2012*
			Market Value (millions of $ - External factor)				
Min	2027.1000	1968.3000	1823.5000	352.2000	1491.4000	158.4000	1258.7000
Max	284167.7000	274380.6000	259757.8000	161185.9000	209379.0000	215269.0000	269511.6000
Mean	63569.0070	70553.4663	64104.3419	43087.8770	64768.5379	64842.1678	68456.2035
Std	58260.8887	63822.2787	60829.7210	45843.0656	64143.9991	61652.8002	67203.4195
			Total Employee Number (Input)				
Min	8250	8000	8260	8013	7628	8736	8283
Max	1800000	1900000	2055000	2100000	2100000	2100000	2200000

[7]Firm's market value can be defined as the price at which a firm would be able to be traded in a competitive auction setting.

[8]According to Tobin (1969) a firm's investment should be positively related to the ratio of its market value to the replacement value of its capital stock.

Mean	131766	141209	145031	149702	147184	149347	151955
Std	203928	215202	229254	233948	232818	233431	242515
				Total Assets (millions of $- Input)			
Min	4404.6000	4703.9000	5220.9000	5023.8000	5902.1000	6488.3000	5785.4000
Max	1494037.0000	1884318.0000	2187631.0000	2175052.0000	2223299.0000	2264909.0000	2265792.0000
Mean	127108.1124	145493.1618	162572.8798	167239.6427	172465.1022	181952.7022	186468.8708
Std	272937.5695	322462.7693	370378.7692	394511.4318	402790.9894	416225.8951	416027.5215
				Total Sales (millions of $ - Output)			
Min	6138.6000	10604.9000	16594.0000	19147.5000	18745.0000	21013.0000	23191.5000
Max	339938.0000	351139.0000	378799.0000	442851.0000	408214.0000	421849.0000	452926.0000
Mean	52522.0382	58200.6697	63838.2652	67255.4034	61670.0697	66758.5180	73186.9584
Std	54785.6955	58053.6867	62289.0881	69282.9998	56656.6833	61885.1947	71270.6594

In this analysis a sample of 89 biggest firms as have been extracted from the Fortune Global 500 companies for the time period of 2006-2012[9] has been used. Table 1 provides a diachronical view of the descriptive statistics of the variables used in the analysis. In order to measure firms' operational efficiency a standard production function based approach has been used as has been applied by several other DEA studies (Seiford and Zhu, 1999; Luo, 2003; Chen, 2004; Kao and Hung, 2007). Therefore, total assets and total employment have been used as inputs whereas total sales volume has been used as our main output variable. Finally, as has been analyzed previously market value has been applied as the exogenous factor explaining firms' operational efficiency levels.

3.2 Empirical results

The global effect of the market value (MV) on firms' efficiency levels $Q = \frac{1+\hat{\Delta}\left(p^{\xi},q^{\xi};0,S_m \middle| \omega\right)}{1+\hat{\Delta}\left(p^{\xi},q^{\xi};0,S_m\right)}$, based on the local linear estimator is presented on Figure 1. Firstly following the relative literature (Racine, 1997, 2008; Racine et al., 2006; Li and Racine, 2007) a nonparametric regression bootstrap based significance test has been applied and in order to find if market value (MV) variable is statistically significant on explaining firms' efficiency variations. A *p-value*=0.0151 has been obtained indicating that MV is statistically significant at the 5% significance level. As can be observed the relationship between firms' market value and their efficiency levels has a “U” shape form. This indicates that for lower market value the firms' inefficiency levels are high. In contrast when firms' market value increases, firms' inefficiency levels are decreasing. The results revealed from the analysis support the literature indicating that there is a strong connection between firm's market value and its efficiency levels.

[9]The sample has been narrowed down to 89 firms since we have a lot of missing values for the examined time period.

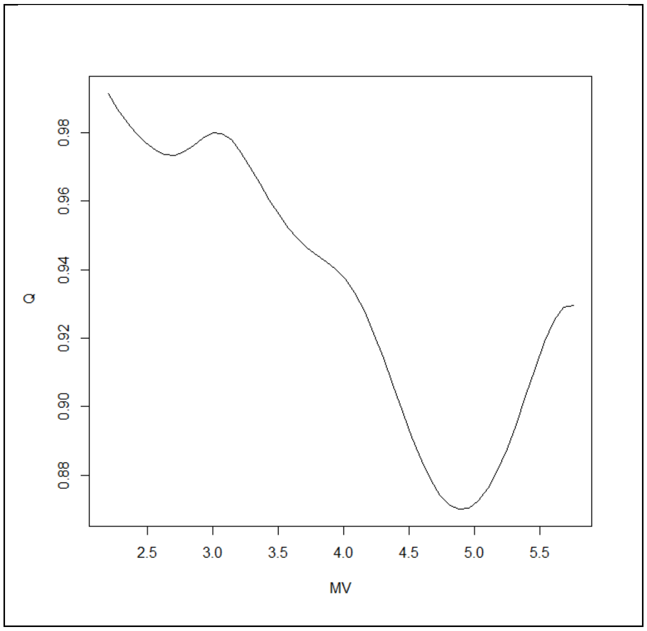

Figure 1: The visualization effect of market value on firms' performance using the local linear estimator.
Note: Firms' market values (MV) are displayed in logarithmic form.

4 Conclusions

This paper provides an innovative extension of the RDM estimator introduced by Silva Portela et al. (2004). By applying the latest developments of the probabilistic characterization of the DDF (Simar and Wilson 2012), the paper introduces an output conditional range directional distance (CRDD) estimator. It is proven that as in the case of the RDM the proposed estimator is translation and units invariance and can handle also negative data. Moreover, since its probabilistic nature can handle the effect of environmental variables without imposing the separability assumption. Finally, an empirical application of the estimator is presented analyzing the effect of firms' market value on their efficiency levels. The results from the nonparametric analysis reveal a "U"- shape relationship indicating that lower market values are associated with firms' lower operating efficiency levels, whereas, higher market values indicate firms' with higher operating efficiency levels.

Appendix A: Proofs of translation and unit invariance of the CRDD estimator.

A1.1 Proof for translation invariance

As in the study by Silva Portela et al. (2004) in order to prove that the model presented in (19), it is assumed that an amount Θ_m is added to each output. Then the constraints become

$$\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}(\eta_\xi q_{\xi m}+\Theta_m)\geq(q_{\xi m}+\Theta_m)+\gamma S_{\xi m}. \quad \text{(A.1)}$$

As can be observed from (A.1) the range improvement does not change with the addition of Θ_m. In addition the left hand side of the restriction can be written as:

$$\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi(q_{\xi m}+\Theta_m)=\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi q_{\xi m}+\Theta_m\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi. \quad \text{(A.2)}$$

Given that under the VRS assumption $\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi=1$, then

$$\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi q_{\xi m}+\Theta_m\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi\geq(q_{\xi m}+\Theta_m)+\gamma S_{\xi m} =>$$

$$\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi q_{\xi m}+\Theta_m\geq(q_{\xi m}+\Theta_m)+\gamma S_{\xi m} =>$$

$$\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi q_{\xi m}\geq q_{\xi m}+\gamma S_{\xi m}. \quad \text{(A.3)}$$

As presented above then the constraints changed with Θ_m reduce to the constraints in (19). As demonstrated here the conditional directional distance estimator can only be translation invariant under the assumption of variable returns to scale.

A1.2 Proof for units invariance

As above let us assume now that we multiply all levels of outputs by Θ_m. The constraints of (19) are modified as:

$$\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi\Theta_m q_{\xi m}\geq\Theta_m q_{\xi m}+\gamma\Theta_m S_{\xi m}. \quad \text{(A.4)}$$

Similarly the constraints in (A.4) reduce to the constraints in (19)

$$\sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi\Theta_m q_{\xi m}\geq\Theta_m q_{\xi m}+\gamma\Theta_m S_{\xi m} => \sum_{\xi=1,\dots,\Xi||\Omega_\xi-\omega|\leq h}\eta_\xi q_{\xi m}\geq q_{\xi m}+\gamma S_{\xi m}. \quad \text{(A.5)}$$

Therefore the solution does not change when the unit of measurement changes.

References

Akther, S., Fukuyama, H., & Weber, W. L. (2013). Estimating two-stage network slacks-based inefficiency: An application to Bangladesh banking. Omega, 41(1), 88-96.

Asmild, M., & Pastor, J. T. (2010). Slack free MEA and RDM with comprehensive efficiency measures. Omega, 38(6), 475-483.

Avkiran, N. K. (2009). Removing the impact of environment with units-invariant efficient frontier analysis: An illustrative case study with intertemporal panel data. Omega, 37(3), 535-544.

Bădin, L., Daraio, C., & Simar, L. (2010). Optimal bandwidth selection for conditional efficiency measures: A data-driven approach. European Journal of Operational Research, 201(2), 633-640.

Barros, C. P., Managi, S., & Matousek, R. (2012). The technical efficiency of the Japanese banks: non-radial directional performance measurement with undesirable output. Omega, 40(1), 1-8.

Bogetoft, P., & Hougaard, J. L. (1999). Efficiency evaluations based on potential (non-proportional) improvements. Journal of Productivity Analysis, 12(3), 233-247.

Cazals, C., Florens, J. P., & Simar, L. (2002). Nonparametric frontier estimation: a robust approach. Journal of econometrics, 106(1), 1-25.

Chambers, R. G., Chung, Y., & Färe, R. (1996). Benefit and distance functions. Journal of Economic Theory, 70(2), 407-419.

Chambers, R. G., Chung, Y., & Färe, R. (1998). Profit, directional distance functions, and Nerlovian efficiency. Journal of Optimization Theory and Applications, 98(2), 351-364.

Chen, Y. (2004). Ranking efficient units in DEA. Omega, 32(3), 213-219.

Chen, Y., Du, J., & Huo, J. (2013). Super-efficiency based on a modified directional distance function. Omega, 41(3), 621-625.

Chung, Y. H., Färe, R., & Grosskopf, S. (1997). Productivity and undesirable outputs: a directional distance function approach Journal of Environmental Management, 51(3), 229-240.

Daraio, C., & Simar, L. (2005). Introducing environmental variables in nonparametric frontier models: a probabilistic approach. Journal of Productivity Analysis, 24(1), 93-121.

Daraio, C., & Simar, L. (2006). A robust nonparametric approach to evaluate and explain the performance of mutual funds. European Journal of Operational Research, 175(1), 516-542.

Daraio, C., & Simar, L. (2007). Conditional nonparametric frontier models for convex and nonconvex technologies: a unifying approach. Journal of Productivity Analysis, 28(1-2), 13-32.

Färe, R., & Grosskopf, S. (2009). A comment on weak disposability in nonparametric production analysis. American Journal of Agricultural Economics, 91(2), 535-538.

Färe, R., & Grosskopf, S. (2013). DEA, directional distance functions and positive, affine data transformation. Omega, 41(1), 28-30.

Färe, R., Grosskopf, S., & Pasurka Jr, C. A. (2007a). Environmental production functions and environmental directional distance functions. Energy, 32(7), 1055-1066.

Färe, R., Grosskopf, S., & Pasurka Jr, C. A. (2007b). Pollution abatement activities and traditional productivity. Ecological Economics, 62(3-4), 673-682.

Färe, R., Grosskopf, S., & Pasurka Jr, C. A. (2010). Toxic releases: an environmental performance index for coal-fired power plants. Energy Economics, 32(1), 158-165.

Färe, R., Grosskopf, S., & Pasurka, C. A. (2006). Social responsibility: US power plants 1985–1998. Journal of Productivity analysis, 26(3), 259-267.

Färe, R., Grosskopf, S., Lovell, C. K., & Pasurka, C. (1989). Multilateral productivity comparisons when some outputs are undesirable: a nonparametric approach. The Review of Economics and Statistics, 90-98.

Farrell, M. J. (1957). The measurement of productive efficiency. Journal of the Royal Statistical Society: Series A (General), 120(3), 253-281.

Hall, P., Racine, J., & Li, Q. (2004). Cross-validation and the estimation of conditional probability densities. Journal of the American Statistical Association, 99(468), 1015-1026.

Hughes, J. P., Lang, W., Moon, C. G., & Pagano, M. S. (1997). Measuring the efficiency of capital allocation in commercial banking. Working paper no. 98-2, Federal Reserve Bank of Philadelphia, Philadelphia, USA.

Jeong, S. O., Park, B. U., & Simar, L. (2010). Nonparametric conditional efficiency measures: asymptotic properties. Annals of Operations Research, 173(1), 105-122.

Johnson, A. L., & Kuosmanen, T. (2011). One-stage estimation of the effects of operational conditions and practices on productive performance: asymptotically normal and efficient, root-n consistent StoNEZD method. Journal of Productivity Analysis, 36(2), 219-230.

Johnson, A. L., & Kuosmanen, T. (2012). One-stage and two-stage DEA estimation of the effects of contextual variables. European Journal of Operational Research, 220(2), 559-570.

Kao, C., & Hung, H. T. (2007). Management performance: An empirical study of the manufacturing companies in Taiwan. Omega, 35(2), 152-160.

Kuosmanen, T. (2005). Weak disposability in nonparametric production analysis with undesirable outputs. American Journal of Agricultural Economics, 87(4), 1077-1082.

Kuosmanen, T. (2012). Stochastic semi-nonparametric frontier estimation of electricity distribution networks: Application of the StoNED method in the Finnish regulatory model. Energy Economics, 34(6), 2189-2199.

Kuosmanen, T., & Matin, R. K. (2011). Duality of weakly disposable technology. Omega, 39(5), 504-512.

Kuosmanen, T., & Podinovski, V. (2009). Weak disposability in nonparametric production analysis: reply to Färe and Grosskopf. American Journal of Agricultural Economics, 91(2), 539-545.

Li, Q., & Racine, J. (2004). Cross-validated local linear nonparametric regression. Statistica Sinica, 485-512.

Li, Q., & Racine, J. S. (2007). Nonparametric econometrics: theory and practice. Princeton University Press, New Jersey.

Luenberger, D. G. (1992). Benefit functions and duality. Journal of Mathematical Economics, 21(5), 461-481.

Luenberger, D. G. (1994). Optimality and the theory of value. Journal of Economic Theory, 63(2), 147-169.

Luo, X. (2003). Evaluating the profitability and marketability efficiency of large banks: An application of data envelopment analysis. Journal of Business Research, 56(8), 627-635.

McConnell, J. J., & Servaes, H. (1990). Additional evidence on equity ownership and corporate value. Journal of Financial Economics, 27(2), 595-612.

Mehran, H. (1995). Executive compensation structure, ownership, and firm performance. Journal of Financial Economics, 38(2), 163-184.

Modigliani, F., & Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. The American Economic Review, 48(3), 261-297.

Morck, R., Shleifer, A., & Vishny, R. W. (1988). Management ownership and market valuation: An empirical analysis. Journal of Financial Economics, 20, 293-315.

Portela, M. S., Thanassoulis, E., & Simpson, G. (2004). Negative data in DEA: A directional distance approach applied to bank branches. Journal of the Operational Research Society, 55(10), 1111-1121.

Racine, J. (1997). Consistent significance testing for nonparametric regression. Journal of Business & Economic Statistics, 15(3), 369-378.

Racine, J. S. (2008). Nonparametric econometrics: A primer. Foundations and Trends® in Econometrics, 3(1), 1-88.

Racine, J. S., Hart, J., & Li, Q. (2006). Testing the significance of categorical predictor variables in nonparametric regression models. Econometric Reviews, 25(4), 523-544.

Racine, J., & Li, Q. (2004). Nonparametric estimation of regression functions with both categorical and continuous data. Journal of Econometrics, 119(1), 99-130.

Ramanathan, R., & Yunfeng, J. (2009). Incorporating cost and environmental factors in quality function deployment using data envelopment analysis. Omega, 37(3), 711-723.

Seiford, L. M., & Zhu, J. (1999). Profitability and marketability of the top 55 US commercial banks. Management Science, 45(9), 1270-1288.

Simar, L., & Vanhems, A. (2012). Probabilistic characterization of directional distances and their robust versions. Journal of Econometrics, 166(2), 342-354.

Simar, L., & Wilson, P. W. (2007). Estimation and inference in two-stage, semi-parametric models of production processes. Journal of Econometrics, 136(1), 31-64.

Tobin, J. (1969). A general equilibrium approach to monetary theory. Journal of Money, Credit and Banking, 1(1), 15-29.

Wang, C. H., Lu, Y. H., Huang, C. W., & Lee, J. Y. (2013). R&D, productivity, and market value: An empirical study from high-technology firms. Omega, 41(1), 143-155.

Zelenyuk, V. (2013). A scale elasticity measure for directional distance function and its dual: Theory and DEA estimation. European Journal of Operational Research, 228(3), 592-600.