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Robustness of Optimal Interest Rate Rules in an Open Economy

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Abstract

This paper studies optimal interest-rate rules that are robust with respect to exogenous shocks in open economies. When derived from closed economy optimization models, theoretical interest-rate rules tend to be more aggressive than empirical rules. We show that accounting for openness in an economy results in optimal rules that correspond more closely to the rules used in practice: the robustly optimal rules derived in an open-economy model exhibit less inertia and sensitivity to variations in macroeconomic variables.

JEL classification numbers: E50, E52, E58.
Keywords: robustness, optimal interest-rate rule, degree of openness.

1 Introduction

Interest rate setting is a central issue of monetary policy conduct. When monetary policy is designed under an uncertain environment, robustness of interest rate rules becomes a fundamental concern. Various methods are proposed in the literature to improve the performance of the interest rate rules under uncertainty. In this paper we consider two important methods: robust control and robustly optimal instrument rules.

Uncertainty is a key element in today's macroeconomics environment and therefore becomes an unavoidable issue for monetary authorities when designing monetary policy. Uncertainty varies greatly in the particular form it takes and in the way in which it affects the economic system. The robust control approach developed by Hansen and Sargent (2003, 2008) seeks monetary policy rules that are robust against model uncertainty. This approach aims at determining policy rules that are able to work reasonably well even when the worst case scenario turns out to be true. See also Dennis *et al*. (2009) for a

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general presentation of this approach. The recent emergence of a literature related to robust control approach has already proved fruitful. For instance, using a macroeconomic model with asset prices, Dai and Spyromitros (2012a) show that a higher central bank preference for robustness implies a more aggressive reaction of the nominal interest rate to the expected future inflation rate and inflation shocks. Tillmann (2014) studies optimal delegation with a misspecified potential output, while Sorge (2013) discuss the same issue with uncertain monetary policy preference. Qin *et al*. (2013) analyze how model uncertainty affects the inflation persistence dynamic. In this literature, a particular attention has been paid to the impact of uncertainty[3] of central bankers' preference for robustness on monetary policy conduct, see for example, Dai and Spyromitros (2010) and (2012b), Qin, *et al*. (2010).

Giannoni and Woodford (2003a, 2003b) propose an alternative method to achieve monetary policy robustness. The method results in so-called robustly optimal instrument rules (referred to, henceforth, ROI rules). As Giannoni and Woodford (2003a) pointed out, the ROI rules are attractive for a number of reasons. First, the rules are robust in the sense that the optimality is independent of the specification of stochastic disturbances, since the rules contain merely target variables. Second, the proposed rules result in determinate equilibriums in which the policy responds to shocks in an optimal manner. Furthermore, if one adopts the "timeless perspective", commitments to such rules do not necessarily imply time inconsistency. Because of these appealing features, the study of optimal monetary policy has been increasingly focused on the ROI rules and a series of similar rules. Aoki and Nikolov (2005) evaluate performances of the ROI rules together with the optimal non-inertial rule and the optimal price-level targeting rule, assuming that the central bank is learning about the parameter values of a simple New Keynesian model. Hansen (2007) examines optimal instrument rules under the imperfect commitment regime. Giannoni and Woodford (2003b) apply the principle of robustly optimal rule to a range of alternative specifications including a simple forward-looking model, a New Keynesian Phillips curve with inflation inertia and a model with sticky prices and wages.

Most of these studies focus on closed economies and few analyses have been conducted under an open economy context. Using a micro-founded New-Keynesian model, the paper attempts to provide insights into the design of optimal interest rate in an open economy. Extending the analysis to open economies has both practical and theoretical significations. Central banks' monetary policy making is taking place under the context of globalization. Different economic entities become more and more interdependent. Policy making that fails to take account of open economy features may lead to undesirable results. Results derived in closed economy models do not necessarily hold in an open economy environment. For instance, Ellison et al. (2007) show that an activist policy which is optimal in a closed economy may be suboptimal in a two-country world. Thus, the extension to the open economy model can be considered as a test of robustness of the ROI rule in an alternative specification.
The contributions of this paper are three-fold: first, we derive a robust optimal instrument

[3]The issue of central bank transparency has been investigated by a large number of studies. Some recent studies are the following: Papadamou *et al*., 2014; Spyromitros, 2014; Dai *et al*., 2015; Demertzis and Hughes Hallett, 2015; James and Lawler, 2015; Papadamou and Arvanitis, 2015; Papadamou *et al*., 2015; Horváth and Vaško, 2016; Papadamou *et al*., 2016.

rule in the sense of Giannoni and Woodford (2003a, 2003b) characterized by open economic features. In spite of the additional uncertainty arising from international economic markets, the proposed rule remains optimal and robust.

Second, we compare the open economy rule with the same type of rule obtained in closed economy model and find the former exhibits less inertia and sensitivity to variations in macroeconomic variables. Theoretically derived optimal interest-rate rules deviate from the rules used in practice such as the Taylor rule. When derived from closed economy optimization models, the rules tend to be more aggressive than actually employed rules (see for example Cateau, 2007). Thus, our finding shows that accounting for openness in an economy results in optimal rules that correspond more closely to the empirical rules.

Finally, we compare the robust control approach with Giannoni and Woodford's (2003a, 2003b) method in terms of the resulting policy rules. Walsh (2004) compares the ROI rule with the rule derived from the robust control method proposed by Hansen and Sargent (2008) and finds that the two seemingly different strands lead to an equivalent result. We reexamine this result in open economy environment and prove that the equivalence between the robust optimal instrument rule and the robust policy derived from the robust control method prevails in the open economic framework.
The remainder of the paper is structured as follows. In section 2, we present the New Keynesian open economy model and introduce an objective function with interest rate target. Section 3 derives the robust optimal interest rule in an open economic environment and discusses its characteristics. Section 4 compares the ROI rule in an open economy with the one of a closed economy, emphasizing on the role of degree of openness. Section 5 seeks to determine an optimal rule by applying robust control approach. Finally, section 6 provides some concluding remarks.

2 A New-Keynesian open economy model

Our analysis is based on an open economy model which is a generalization of the canonical New-Keynesian model for a closed economy as in Gal and Monacelli (2005), Clarida *et al.* (2002) and Walsh (2003, ch. 6.5). The basic assumptions of the model are as follows. The world is composed of two economies: a small open economy and a relatively closed great economy. The two economies share the same preferences and technologies and produce exchangeable goods. In the domestic country, the firms produce the domestic goods, and the households consume the domestic and imported goods. The small open economy is characterized by the following equations:

$$\pi_t = \beta E_t \pi_{t+1} + k x_t + \mu e_t + \varepsilon_t \tag{1}$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) - \varphi(E_t e_{t+1} - e_t) + u_t \tag{2}$$

$$e_t = E_t e_{t+1} - (i_t - E_t \pi_{t+1}) + \varphi_t \ . \tag{3}$$

The parameter β is the discount rate; π_t, the inflation rate in the domestic goods sector; x_t, the output gap in the domestic economy; i_t, nominal interest rate and e_t, the real exchange rate. And $E_t\pi_{t+1}$, $E_t x_{t+1}$ and $E_t e_{t+1}$ are respectively the expectations of domestic inflation, output gap and exchange rate for the period $t + 1$, based on the information available on period t.

The real exchange rate is defined in terms of the domestic price level

$$e_t = s_t + p_{f,t} - p_t, \quad (4)$$

where s_t denotes the nominal exchange rate, $p_{f,t}$ and p_t are respectively the foreign and domestic price level.

Equation (1) corresponds to a Keynesian version of the Phillips curve for an open economy where the domestic inflation rate depends on the expected future inflation and, through the marginal cost, on the output gap and the exchange rate. A variation in the real exchange rate will affect the Consumer Price Index as well as the expectations on nominal wages which will in turn modify the labor supply and marginal cost as a result.

Equation (2) describes an IS curve in open economy where the current output gap varies positively with the expected output gap and negatively with the real interest rate and the difference between the expected exchange rate and its current value. Real exchange rate affects the output gap, since household's consumption is partly satisfied by imported goods. The demand shock reflects changes in the flexible price output level, or, put differently, variations in the natural level of the real interest rate.

The last component of the model is a condition of interest rate parity given by equation (3), according to which a real depreciation is related to the actual interest rate differential between domestic and foreign economies. The disturbance affecting the exchange rate, φ_t, reflects the premium paid by domestic households for holding foreign assets.

The model parameters k, ϕ, μ, σ are all positive and depend on the underlying microeconomic foundations[4].

$$k = \frac{(\hat{\sigma}+\eta)(1-\gamma)(1-\beta\gamma)}{\gamma}; \quad (5)$$

$$\mu = \frac{\alpha(1-\gamma)(1-\beta\gamma)}{\gamma}; \quad (6)$$

$$\sigma = \frac{\hat{\sigma}}{1-\alpha}; \quad (7)$$

$$\varphi = (2-\alpha)\alpha\delta - \frac{\alpha(1-\alpha)}{\hat{\sigma}}, \quad (8)$$

where $\hat{\sigma}$ denotes the intertemporal elasticity of substitution, γ the probability for a given firm to keep its price unchanged, η the elasticity of labor supply, α the foreign goods share in total consumption and δ the elasticity of substitution between foreign and domestic goods. Notice as $\alpha = 0$ (or $\mu = \phi = 0$), our model reduces to a standard closed economy

[4] The way we obtain equations (5) to (8) is available upon request.

similar to those analyzed by Kerr and King (1996) or Clarida *et al*. (1999).
All perturbations are treated as exogenous autoregressive processes:

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \xi_t, \qquad 0 < \rho_\varepsilon < 1; \tag{9}$$

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \varsigma_t, \qquad 0 < \rho_\upsilon < 1; \tag{10}$$

$$\varphi_t = \rho_\varphi \varphi_{t-1} + \zeta_t, \qquad 0 < \rho_\varphi < 1; \tag{11}$$

where ξ_t, ς_t and ζ_t are i.i.d sequence of random variables with mean zero and variance 1.

The central bank's objective is to minimize the fluctuation of inflation, output gap and nominal interest rate, which is specified as

$$V = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_i (i_{t+i} - i^*)^2\right], \tag{12}$$

where i^* represents the interest rate target and λ_x, λ_i denote respectively the relative weights that the central bank assigns to the output gap and interest rate stabilization objectives. Woodford (2003) shows the above loss function can be interpreted as a second order approximation to the representative agent's utility function and that the relative weights λ_x and λ_i can be determined by the underlying structural parameters under certain conditions.

A novel feature of this objective function is the presence of the third target, nominal interest rate, which plays a crucial role in the derivation of the optimal monetary policy rule. Cukierman (2001) justifies the penalization of the interest rate variability by underlying that the latter increases the risk of financial crises by interfering with the ability of private banks to efficiently hedge their assets and liabilities. Indeed, the variability of short-term nominal interest rates is taken into account by much empirical studies of optimal monetary policy, when evaluating the performance of a policy rule. Furthermore, several recent empirical works find that a significant weight attached to interest rate stabilization objective in the loss function is necessary to obtain an optimal rule corresponding to historical evaluations, especially in backward-looking models. It will be shown below that introducing interest rate stabilization policy objective plays a crucial role in the determination of the optimal monetary policy rule.

3 Interest rate rule in an open economy framework

We will assume throughout that a credible commitment regime is plausible and the monetary authority is able to commit to certain policy rule. Under such assumption, the main assignment of monetary policy making is to seek interest rate rules which work well under the central bank's reference model.
Applying the general method proposed in Svensson and Woodford (2004) and Giannoni and Woodford (2003a, 2003b), we attempt to derive an open economic version of a robust

optimal instrument rule. One important characteristic of Giannoni and Woodfords' dynamic optimization-based method for solving for the robust policy is the so-called "timeless perspective". The timelessly optimal policy is the policy that the monetary authority would have decided upon for the current period had such a decision been taking infinitely far in the past. In such context, the central bank is not bound to some given rule that may reveal unpleasant in the future but rather commits to a criterion by which to assess optimality at each date. Since the central bank's model is expected to guide its decision in the future as well, the rule currently adopted should conform to future behavior. Therefore, justified from a "timeless perspective", committing to a ROI rule does not involve a time inconsistency problem.

Given the model described by equations (1)-(3), the Lagrangian function of the decision makers's problem can be written:

$$L_2 = E_t \sum_{i=0}^{\infty} \beta^i \, [\frac{1}{2}\left(\pi_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_i (i_{t+i} - i^*)^2\right)$$

$$+ s_{1_{t+i}}(\pi_{t+i} - \beta\pi_{t+1+i} - kx_{t+i} - \mu e_{t+i})$$

$$+ s_{2_{t+i}}\left(x_{t+i} - x_{t+i+1} + \frac{1}{\sigma}(i_{t+1} - \pi_{t+i+1}) + \varphi(e_{t+i+1} - e_{t+1})\right)$$

$$+ s_{3_{t+i}}(e_{t+i} - e_{t+i+1} + i_{t+i} - \pi_{t+i+1})] \,. \tag{13}$$

With the traditional optimization method solving for monetary policy under commitment, the multipliers $s_{ct\text{-}i}$ are generally zeroed out because no previous commitment has been made until the first period. While in the context of a "timeless" perspective, one can suppose that a commitment regime has already been applied in the pass and is available in current and future periods. Hence, when forming expectations in period *t*, one should take account of the information available in period *t*- 1. Assuming in particular that the optimal commitment policy has been adopted at least since period *t* - 2 and taking first order conditions with respect to π_t, x_t, i_t and e_t yield the following optimality conditions:

$$\pi_t + s_{1_t} - s_{1_{t-1}} - \frac{1}{\beta\sigma} s_{2_{t-1}} - \frac{1}{\beta} s_{3_{t-1}} = 0, \tag{14}$$

$$\lambda_x x_t - k s_{1_t} + s_{2_t} - \frac{1}{\beta} s_{2_{t-1}} = 0, \tag{15}$$

$$\lambda_i (i_t - i^*) + \frac{1}{\sigma} s_{2_t} + s_{3_t} = 0, \tag{16}$$

$$\mu s_{1_t} + \varphi s_{2_t} - s_{3_t} - \frac{\varphi}{\beta} s_{2_{t-1}} + \frac{1}{\beta} s_{3_{t-1}} = 0. \tag{17}$$

The objective in this stage consists of deriving an expression of interest rate from these optimality conditions. Combining equations (14), (15) and (16), we obtain the following expression:

$$-\frac{\lambda_i}{\beta}(i_{t-1}-i^*)+A\left(s_{2_t}-\frac{1}{\beta}s_{2_{t-1}}\right)+\mu s_{1_t}+\lambda_i(i_t-i^*)=0\ . \qquad (18)$$

Inserting the solution for $s_{2_t}-\frac{1}{\beta}s_{2_{t-1}}$into (15) yields

$$(\mu+Ak)s_{1_t}=\lambda_\chi Ax_t+\frac{\lambda_i}{\beta}(i_{t-1}-i^*)-\lambda_i(i_t-i^*), \qquad (19)$$

where $A=(\frac{1}{\sigma}+\varphi)$and according to which $s_{1_{t-1}}$ can be written as:

$$(\mu+Ak)s_{1_{t-1}}=\lambda_x Ax_{t-1}+\frac{\lambda_i}{\beta}(i_{t-2}-i^*)-\lambda_i(i_{t-1}-i^*). \qquad (20)$$

Rewriting (16) for the period *t* - 1 yields:

$$\lambda_i(i_{t-1}-i^*)+\frac{1}{\sigma}s_{2_{t-1}}+s_{3_{t-1}}=0. \qquad (21)$$

Inserting it into equation (14) gives

$$s_{1_t}-s_{1_{t-1}}=-\pi_t-\frac{\lambda_i}{\beta}(i_{t-1}-i^*). \qquad (22)$$

Then by replacing equations (19) and (20) into equation (21) and rearranging the terms, we obtain the optimal interest rate expressed in terms of the targeting variables:

$$i_t=\ -\left(\frac{k}{\beta\sigma}+\frac{D}{\beta}\right)i^*+\left(1+\frac{k}{\beta\sigma}+\frac{1}{\beta}+\frac{D}{\beta}\right)i_{t-1}-\frac{1}{\beta}i_{t-2}\qquad +$$

$$\left(\frac{k}{\sigma\lambda_i}+\frac{D}{\lambda_i}\right)\pi_t+\left(\frac{\lambda x}{\sigma\lambda_i}+\varphi\right)(x_t-x_{t-1}) \qquad (23)$$

where all parameters are positive and $D=\mu+\phi k$.

This rule can be rewritten in a more general form:

$$i_t=(1-n_1)i^*+n_1 i_{t-1}+n_2\Delta i_{t-1}+\gamma_\pi\pi_t+\gamma_x\Delta x_t, \qquad (24)$$

where $n_1=\left(1+\frac{k}{\beta\sigma}+\frac{D}{\beta}\right)>1, n_2=\frac{1}{\beta}>1$;

$\gamma_\pi = \left(\frac{k}{\sigma\lambda_i} + \frac{D}{\lambda_i}\right), \gamma_x = (\frac{\lambda x}{\sigma\lambda_i} + \varphi)$.

As such optimal interest rate is determined by its target level, its first and second lags, the inflation rate and the differenced output gap. On the other hand, real shocks have no effect on its value. Giannoni and Woodford (2003a) have shown that a commitment to such rule leads to a determinate rational expectations equilibrium. Coefficient (1- n_1) measures the weight attributed to the interest rate adjustment with respect to its targeted level. Note that the coefficients of interest rate, its lags and its targeted level sum to one. This means that the interest rate adjustment towards its reference value is a weighted mean of its target level and its past values. Coefficients γ_π and γ_x respectively describe the interest rate reactions to the inflation and to the differenced output gap.
A remarkable point is that the open economic version of the optimal policy rule does not involve real exchange rate. Whereas, the open economy features are reflected in the coefficient *D* composed by those parameters that determine the impact of variation of exchange rate on inflation (μ) and output (ϕ). Indeed, these parameters are functions of the openness degree of the economy, α, measured by the total share of imported goods in the domestic consumption. Therefore, the difference between an open economy ROI rule and the rule in closed economy depends on the key underlying parameter α.

As demonstrated in the above analysis, the determination of the optimal rule did not involve any assumption concerning the disturbances processes. Indeed, the optimal property of the rule is independent of the nature of perturbations. As shown in Giannoni and Woodford (2003a), a commitment to this rule implies the same optimal impulse responses than those put forward by Woodford (1999) in special cases of simulated perturbations processes. Moreover, it implies as well that the response will be optimal whatever the nature of the perturbations affecting interest rate or of the cost pushed shocks. In addition to the literature, our analysis demonstrates the open economy aspects do not alter the optimality and robustness of the rule. Taking account of the role of the exchange rate in the model does not affect the component of the ROI rule. Furthermore, such robustness proves to be a strong advantage as regards its adoption as a practical guide for the conduct of monetary policy.

It is important however to show that the equation (23) is not the sole optimal instrument rule; this rule is not even the only rule to be robustly optimal in the sense discussed above, as pointed out by Giannoni and Woodford (2003b). They demonstrate, by appropriate variable substitutions and using one of the structural equations (1)-(3), one can derive alternative forms of the ROI which are compatible with optimal responses to the perturbations. Those alternative rules will however not be direct ones in the sense that they will induce feedbacks involving expected future disturbances. As a special case, such a rule can be obtained by imposing the necessary assumptions on the statistical properties of the model perturbations that will allow us to get rid of the disturbance terms.

4 Comparing with the closed economy case

In a closed economy, the robustly optimal interest rate rule can be obtained by setting α and henceforth μ, ϕ, D to zero. It takes the form:

$$i_t = -\frac{k}{\beta\hat{\sigma}} i^* + \left(1 + \frac{k}{\beta\hat{\sigma}} + \frac{1}{\beta}\right) i_{t-1} - \frac{1}{\beta} i_{t-2} + \left(\frac{k}{\hat{\sigma}\lambda_i}\right) \pi_t + \frac{\lambda x}{\hat{\sigma}\lambda_i} (x_t - x_{t-1}) \qquad (25)$$

It corresponds to the robustly optimal instrument rule (ROI) in a closed economy studied by Giannoni and Woodford (2003b). The rule of closed economy can thus be viewed as a special case of the rule deduced in open economy given in (23). Note that (25) can be written in the general formulation with

$$n_1{}' = \left(1 + \frac{k}{\beta\hat{\sigma}}\right) > 1\,,;' = \frac{1}{\beta} > 1\,; \gamma_\pi{}' = \frac{k}{\hat{\sigma}\lambda_i}\ ;\ \gamma_x{}' = \frac{\lambda\chi}{\hat{\sigma}\lambda_i}.$$

Both closed and open economy rules involve some identical features. First, they are both robust to exogenous shocks as coefficients in equations (23) and (25) are influenced neither by the periodical correlations of the shocks coefficients nor by the innovations variances. They can thus be considered as robust as regards misspecifications of these aspects of the model. Second, despite the role that exchange rate plays in an open economy, the ROI rule is independent of this indicator. In other words, both rules contain the same determinants. However, it is noteworthy that the coefficients premultiplied each determinant are different for the two rules.

The following expressions allow us to compare the coefficients in the two rules:

$$\Delta n_1 = n_1 - n_1{}' = \frac{k}{\beta}\left(\varphi + \frac{u}{k} - \frac{\alpha}{\hat{\sigma}}\right);$$

$$\Delta n_2 = n_2 - n_2{}' = 0;$$

$$\Delta\gamma_\pi = \gamma_\pi - \gamma_\pi{}' = \frac{u+\varphi k}{\lambda_i} - \frac{\alpha k}{\hat{\sigma}\lambda_i};$$

$$\Delta\gamma_x = \gamma_x - \gamma_x{}' = \varphi - \frac{\alpha\lambda_x}{\hat{\sigma}\lambda_i}.$$

Apparently, the coefficient premultiplied by the variation of lag of interest rate $\Delta i_{t\text{-}1}$ remains the same in both cases. By contrast, the other three coefficients differ from the cases of closed economy when taking into account the open economic features. The discrepancy of respective coefficient in two rules depends upon the structural parameters, especially, the degree of openness measured by α. It is thus of interest to understand the role of this parameter in the determination of a ROI rule in an open economy. Or, put differently, the question is to which extent it makes an open economy rule to differ from a rule in a closed economy. In particular, does taking account of the openness of the economy increase or dampen the interest rate response to shocks?
To investigate the impact of the degree of openness α on the determination of a ROI rule

in an open economy, we calibrate the coefficients of the optimal instrument rule with the baseline parameters reported in Table 1. Following much of the calibrations of New Keynesian model, we assume the discounted rate β = 0.99. The choice of $\widehat{\sigma}$ = 0.16 and the Calvo parameter of price stickiness γ = 0.66 is based on estimates of Rotemberg and Woodford (1998). The central bank's preference parameter λ_x and λ_i are set as in Giannoni and Woodford (2003b). Gal and Monacelli (2005) calibrate the labour supply elasticity, n,to be 0.3.

Table 1: Calibrated parameter values

Structural parameters					Loss function	
β	$\widehat{\sigma}$	γ	n	δ	λ_x	λ_i
0.99	0.16	0.66	0.3	0.5	0.048	0.236

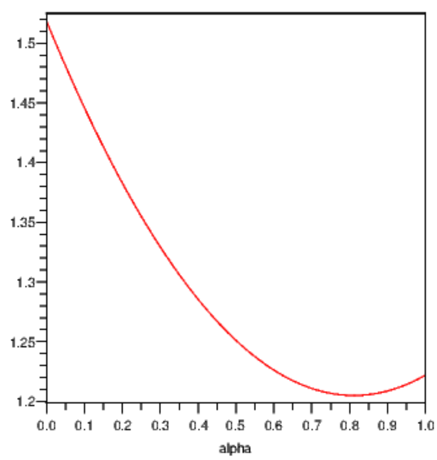

Figure 1: Coefficient n_1 of the optimal instrument rule as functions of α

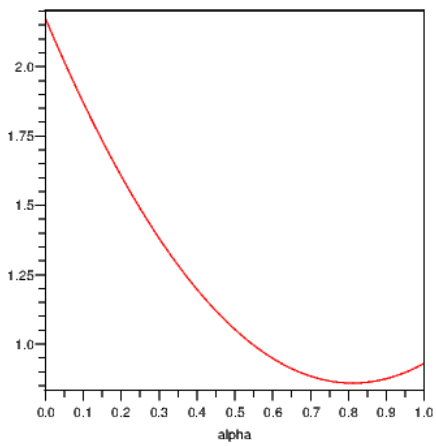

Figure 2: Coefficient γ_π of the optimal instrument rule as functions of α

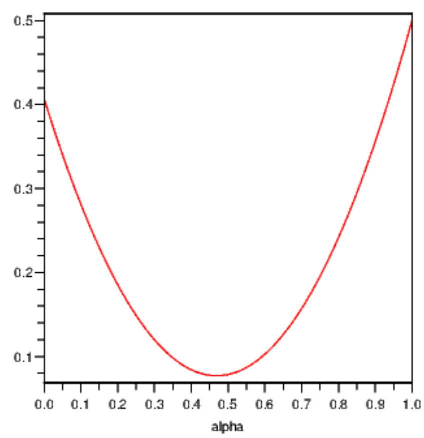

Figure 3: Coefficient γ_X of the optimal instrument rule as functions of α

We calculate first the implied model parameters k, u and ϕ basing on these underlying structural parameters. Then, the numerical values of the coefficients are plotted for alternative values of the degree of openness α ranging from zero to one. As $\alpha = 0$, the point at each curve corresponds to the coefficient in the closed economy rule. As such, the evolution of these curves will enable us to compare the coefficients of the closed economy rule and those of the open economy.

Prior to compare the difference between the rule of open economy and the closed economy one, it is suitable to discuss the signification of these coefficients in the robust optimal rule. The coefficients n_1, n_2 and γ_x denote respectively on the history-dependence of the optimal rule on the lagged interest rate, the lagged rate of increase in the interest rate and the lagged output gap. Due to these non zero weights, Giannoni and Woodford (2003b) qualify thus the ROI rule as super-inertial.

Figure 1 displays the evolution of the coefficient characterizing the interest rate inertia, n_1 as a function of the degree of openness α. The coefficients in an open economy rule are smaller than that of a closed economy. Furthermore, n_1 is decreasing in the degree of openness, as the latter is inferior to a certain value (in this case about 0.8). Since the weight n_1 affects the responses of interest rate both to its target and to the lagged interest rate of the previous period, this result has two effects. On the one hand, it indicates that the optimal interest rate respond less to its target, while the share of foreign goods in the consumption increases. On the other hand, it implies that the intrinsic inertia in the dynamic of the interest rate is alleviated as the economy becomes more open.

From Figure 2, one observes the similar characteristic in the evolution of the weight on current inflation rate γ_π with an increasing degree of openness. It indicates that the increase of interest rate in response to a higher projected inflation rate is attenuated by taking into account the openness of the economy.

Figure 3 shows the changes in the weight on the output gap as a function of α. It is noteworthy what imparts in the optimal interest rate determination is the output gap relative to the previous period's output gap, rather than the absolute level of the projected gap. For low values of α, increases in this parameter dampen the optimal weight on the changes in the output gap projection. In this case, the optimal interest rate rule becomes less backward-looking. Whereas, for high values of α, the weight on the variation of the output gap increases with α. However, it is worth noting that, even in this case, the coefficients γ_x in open economy rules are generally lower than the closed economy one, unless as the degree of openness close to 1.

Although we leave the values of α vary from zero to one, it does not imply that extremely high values of α, such as $\alpha \rightarrow 1$, are realist and representative. In most of the empirical studies, the calibrated values of α remain rather low. The historical average of the share of total imports in GDP for the euro area is about 0.15 (see Smets and Wouters, 2002). Gal and Monacelli (2005) set a baseline value for the degree of openness as 0.4, which corresponds roughly to the import/GDP ratio in Canada. Conforming to the low values of α supported by empirical researches, we can thus conclude the robust optimal interest rate setting is attenuated, taking account of the open economy features. This result is in sharp contrast to the closed economy result which recommends activist policy in order to be robust against uncertainty (see, for example, Giannoni, 2002; Söderström, 2002).

Furthermore, this finding provides a novel argument to reconcile the discrepancy between optimal interest rate rule and empirical rules. In the standard forward-looking models of the recent literature, theoretical optimal monetary policy rules imply much higher inertia of interest rates than estimated historical policy rules. For instance, Giannoni and

Woodford (2003b) estimate the coefficients of the optimal instrument rule as n_1= 1.15, n_2 = 1.01, γ_π = 0.64 and γ_x = 0.33, while the coefficients of the Fed reaction function of similar form estimated by Judd and Rudebusch (1998) are n_1 = 0.73, n_2 = 0.43, γ_π = 0.42 and γ_x = 0.30. According to our finding, as the degree of openness increase, the coefficients premultiplying the lag variables decline. It implies that taking account of the open economy aspect can alleviate this discrepancy between theoretical rules and empirical historical policy rules.

5 Robust control in a timeless perspective

In this section, basing on the same model, we apply the robust control technique to derive an optimal interest rate rule and compare the rule with the one obtain by using Giannoni and Woodford's method. Walsh (2004) shows that the two methods, while seeming quite different, lead to the same robust monetary policy rule. The equivalence between two rules has been proved in a standard and an augmented New Keynesian model with inflation inertia. With the extension of the analysis framework to open economy, it is of interest to investigate the equivalence once again.

The robust control approach developed by Hansen and Sargent (2003, 2008) assumes that the central bank's reference model may be misspecified and then differs from the real economic system. Under this assumption, the central bank takes into account possible model misspecifications and attempts to define monetary policy rules which are robust against the worst scenario. Following Hansen and Sargent (2008, chapter 16), we assume a Stackelberg game where the central bank acts as a Stackelberg leader against a fictitious evil agent who control the model misspecification. The central bank commits to a policy rule taking into account its ability of implementing and the worst case beliefs of the evil agent through its choice of misspecification. Compared to the approach proposed by Hansen and Sargent, a novelty in our paper is that we integrate the "timeless perspective" to the robust Stackelberg problem assuming that the policy commitment has taken place before *t* = 0.

In this Stackelberg problem, the central bank attempts to define an interest rate rule to minimize the losses given the exogenous shocks. Then, given the central bank's decision: the setting of i_t, the evil agent sets ω_{t+1} to maximize the loss function. We assume the central bank and the evil agent share (1)-(3) as their reference model and the following disturbance processes

$$\varepsilon_{t+1} = \rho_\varepsilon \varepsilon_t + \xi_{t+1} + \omega_{1_{t+i+1}}, \qquad 0 < \rho_\varepsilon < 1; \tag{26}$$

$$\upsilon_{t+1} = \rho_\upsilon \upsilon_t + \varsigma_{t+1} + \omega_{2_{t+i+1}}, \qquad 0 < \rho_\upsilon < 1; \tag{27}$$

$$\varphi_{t+1} = \rho_\varphi \varphi_t + \zeta_{t+1} + \omega_{3_{t+i+1}}, \qquad 0 < \rho_\varphi < 1 \tag{28}$$

where $\omega_{1_{t+i+1}}$, $\omega_{2_{t+i+1}}$, and $\omega_{3_{t+i+1}}$represent the misspecifications perturbing the dynamics of the model and satisfy the following constraints:

$\sum_{i=0}^{\infty} \beta^i \omega_{1}{}_{t+1}^2 \leq \Xi_1, \Xi_1 > 0;$

$\sum_{i=0}^{\infty} \beta^i \omega_{2}{}_{t+1}^2 \leq \Xi_2, \Xi_2 > 0\ ;$

$\sum_{i=0}^{\infty} \beta^i \omega_{3}{}_{t+1}^2 \leq \Xi_3, \Xi_3 > 0\ .$

As $\Xi_c \neq 0,\ c = 1,2,3$ both players in the Stackelberg game are concerned by model misspecification. Thus, the multiplier version of the robust Stackelberg problem with an open economy New-Keynesian model takes the form:

$$\min_i \quad \max_\omega E_t^{rc} \sum_{i=0}^{\infty} \beta^i \,[\tfrac{1}{2}\left(\pi_{t+i}^2 + \lambda_\chi x_{t+i}^2 + \lambda_i (i_{t+1} - i^*)^2\right) - \tfrac{1}{2}\beta\theta\left(\omega_{1}{}_{t+i+1}^2 + \omega_{2}{}_{t+i+1}^2 + \omega_{3}{}_{t+i+1}^2\right) + s_{1_{t+1}}(\pi_{t+i} - \beta\pi_{t+1+i} - kx_{t+i} - \mu e_{t+i} - \varepsilon_{t+i}) + s_{2_{t+1}}\left(x_{t+i} - x_{t+i+1} + \tfrac{1}{\sigma}(i_{t+1} - \pi_{t+1+i}) + \varphi(e_{t+1+i} - e_{t+i}) - \upsilon_{t+i}\right) + s_{3_{t+1}}(e_{t+i} - e_{t+1+i} + i_{t+1} - \pi_{t+1+i} - \varphi_{t+i}) + s_{4_{t+1}}\left(\rho_\varepsilon \varepsilon_{t+i} + \xi_{t+i} + \omega_{1_{t+i+1}} - \varepsilon_{t+1+i}\right) + s_{5_{t+1}}\left(\rho_\upsilon \upsilon_{t+i} + \varsigma_{t+i} + \omega_{2_{t+i+1}} - \upsilon_{t+1+i}\right) + s_{6_{t+1}}(\rho_\varphi \varphi_{t+i} + \zeta_{t+i} + \omega_{3_{t+i+1}} - \varphi_{t+1+i} \qquad (29)$$

where $s_{ct} + i, c = 1 \ldots 6$ are the Lagrangian multipliers.

Adapting always the "timeless perspective" and taking the first order conditions with respect to π_t, x_t, ε_t , υ_t, φ_t, ω_{1t} , ω_{2t} et ω_{3t} yields the following results:

$$\pi_t + s_{1_t} - s_{1_{t-1}} - \frac{1}{\beta\sigma} s_{2_{t-1}} - \frac{1}{\beta} s_{2_{t-1}} = 0 \qquad (30)$$

$$\lambda_x x_t - k s_{1_t} + s_{2_t} - \frac{1}{\beta} s_{2_{t-1}} = 0 \qquad (31)$$

$$\lambda_i (i_t - i^*) + \frac{1}{\sigma} s_{2_t} + s_{3_t} = 0 \qquad (32)$$

$$\mu s_{1_t} + \varphi s_{2_t} - s_{3_t} - \frac{\varphi}{\beta} s_{2_{t-1}} + \frac{1}{\beta} s_{3_{t-1}} = 0 \qquad (33)$$

$$-s_{1_t} + \rho_\varepsilon s_{4_t} - \frac{1}{\beta} s_{4_{t-1}} = 0 \qquad (34)$$

$$-s_{2_t} + \rho_\upsilon s_{5_t} - \frac{1}{\beta} s_{5_{t-1}} = 0 \qquad (35)$$

$$-s_{3_t} + \rho_\varphi s_{6_t} - \frac{1}{\beta} s_{6_{t-1}} = 0 \tag{36}$$

$$-\theta\beta\omega_{1_{t+1}} + s_{4_t} = 0 \tag{37}$$

$$-\theta\beta\omega_{2_{t+1}} + s_{5_t} = 0 \tag{38}$$

$$-\theta\beta\omega_{3_{t+1}} + s_{6_t} = 0 \tag{39}$$

Apparently, equations (30)-(33) are identical to the optimality conditions given by (14)-(17). Then, by the same manipulation, we can obtain the following ROI rule:

$$i_t = -\left(\frac{k}{\beta\sigma} + \frac{D}{\beta}\right) i^* + \left(1 + \frac{1}{\beta\sigma} + \frac{1}{\beta} + \frac{D}{\beta}\right) i_{t-1} - \frac{1}{\beta} i_{t-2} + \left(\frac{k}{\sigma\lambda_i} + \frac{D}{\lambda_i}\right) \pi_t + \left(\frac{\lambda_x}{\sigma\lambda_i} + \varphi\right)(x_t - x_{t-1}) \tag{40}$$

The interest rate rule resulting from the robust control approach is equivalent to the optimal robust instrument rule which we analyzed in theprevious subsection. It is worth to note that the coefficients of this rule do not contain the parameter θ which represents the degree of the central bank's desire for robustness. Despite the different manner of modeling and anticipations formation, Hansen and Sargent's robust control approach and Giannoni and Woodford's standard approach induce the same robust optimal instrument rule.

However, in spite of the equivalent results derived from the two approaches, the economic behaviors described in the two model specifications are different. The difference stems from the way in which the expectations are made in two cases. In the optimization approach suggested by Giannoni and Woodford (2003a, 2003b), the exogenous shocks are assumed to follow an auto-regressive process. While, in the environment of robust control, anticipations are based on a distorted version of the disturbance processes.

Then, the arising question is, since the economic behaviors reflected in the two methods are different, how can they lead to identical policy rules? A ROI rule is robust to the exogenous disturbances generating process. On the other hand, the robust control approach seeks to anchor against the misspecification characterizing the evolutions of the state variables. Nevertheless, in our New Keynesian setting up, the state variables are just the exogenous disturbances. Hence, the two seemingly independent strands are designed to be robust with respect to the same source of specification errors.

6 Concluding Remarks

This paper discusses the robustness of optimal monetary policy rules based on a New-Keynesian open economy model with microeconomic foundations. First, assuming a "timeless" commitment mechanism, we obtained an open economic version of robustly optimal instrument interest rule in the sense of Giannoni and Woodford (2003a, 2003b). It is shown that this type of rules involving the optimal response of interest rate is robust with respect to exogenous disturbances process. In particular, the open economy features are reflected in the coefficient of the rules, though determination of the rule is independent of the exchange rate.

Then comparing the open economy rule with the rule derived from a closed economy model, we find that the former is less aggressive than the latter, reflected by lower coefficients of the deviation of the target variables in an open economy rule. Moreover, it is possible to obtain non-super-inertial rules, as the weights on the lag variable is decreasing in the degree of openness. The implication is that taking account of open economy aspects leads to attenuation and alleviates the super-inertia in the optimal rule.

Finally, it is shown that Giannoni and Woodford's (2003a, 2003b) method and the robust control of Hansen and Sargent both lead to the same optimal rule. This result generalizes the equivalence found by Walsh (2004) in a New Keynesian closed economy model to an open economy framework.

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